



PSAC 2017

Grade 6 Mathematics

Subject Code: P120

Examiners' Report

MAURITIUS EXAMINATIONS SYNDICATE

April 2018

MATHEMATICS

(Subject Code No. 120)

Background

The Primary School Achievement Certificate (PSAC) Assessment in Mathematics was introduced for the first time in 2017 in the context of the implementation of the Nine Year Continuous Basic Education (NYCBE) Reform.

Re-designed to meet the overarching aims of the reform, the Mathematics assessment paper serves to measure pupils' achievement standards in the acquisition of mathematical knowledge and understanding effectively, as well as the development of mathematical skills, at the end of the primary cycle. It is aimed at providing an engaging and meaningful assessment experience to learners so they feel confident to progress to Grade 7. More specifically, it has been developed with the intention to ensure that learners of diverse abilities attain a sound mathematical base to be able to sustain future learning and be successful. For this reason, the new mathematics assessment paper places a heightened emphasis on learners' acquisition of conceptual understanding, problem-solving skills and logical thinking skills.

Learners are assessed on three Assessment Objectives (AOs) as follows:

- **Knowledge and comprehension (40 %)** – questions or items developed under this assessment objective are aimed at revealing learners' ability to *'recall specific mathematical facts, concepts, rules and formulae, represent simple mathematical statements or information; perform simple mathematical operations and routine procedures'* (Annual Programme for the Primary School Achievement Certificate (PSAC) Assessment 2017, p. 20). In 2017, these were assessed in **Questions 1-3, 6-14, 16, 18-21, 23, 25-27, 29, 32, and 36.**
- **Application (40%)** – questions specifically developed to provide learners with the opportunities to demonstrate their ability to *'identify and apply mathematical concepts, rules and formulae, skills and techniques to solve familiar problems in Mathematics'*

(Annual Programme for the Primary School Achievement Certificate (PSAC) Assessment 2017, p. 20). **Questions 4, 5, 15, 17, 24, 30, 31, 33-35 and 40** in the Mathematics paper of 2017 are examples of such types of questions.

- **Analysis (20%)** – questions under this assessment objective seek to indicate whether learners can *‘break down and interpret multi-faceted information and data into their component parts; recognise and use unstated mathematical assumptions in problem solving; formulate appropriate strategies to solve non-routine problems’* (Annual Programme for the Primary School Achievement Certificate (PSAC) Assessment 2017, p. 20). Questions that served this purpose in 2017 included **Questions 22, 41 and 45**.

Notwithstanding the fact that it is difficult to delineate questions as purely assessing knowledge and comprehension or application or analysis, a few questions in the paper integrated two or more assessment objectives, such as **Questions 28, 37-39, 41-45**.

Thus, the PSAC Assessment in Mathematics sets out to present a mix of basic, average and engaging questions to enable learners of diverse abilities to demonstrate what they can and what they cannot do in respect of the learning outcomes and the assessment objectives they are expected to achieve at the end of Grade 6.

In response to the need to equip learners with the necessary 21st century skills and, in line with the specifications of the *National Curriculum Framework Grades 1-6*, the Mathematics assessment paper also sets forth to reflect the extent of pupils’ acquisition of the eight components of mathematical proficiency (see table below).

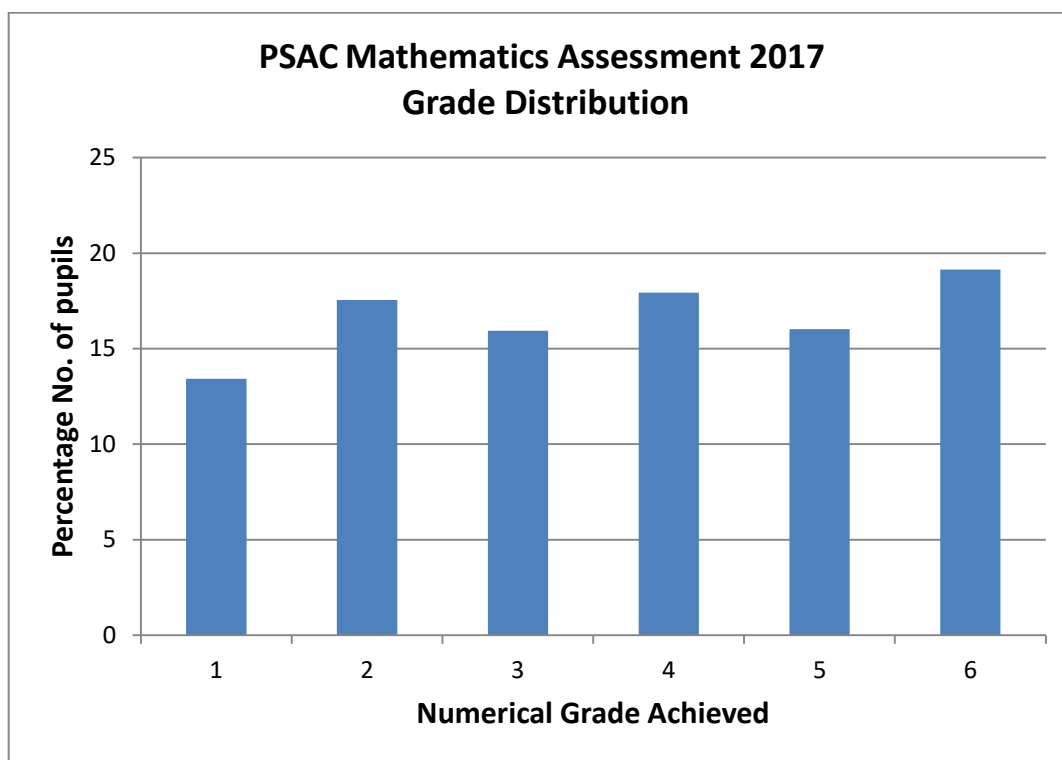
In this way, the PSAC Mathematics assessment paper can be regarded as the product of a matrix of three main interconnected elements: the learning outcomes, the assessment objectives and the components of mathematical proficiency.

Components	Learning Aims
Representation	<i>Use and interpret</i> illustrations of mathematical objects such as graphs, tables, pictorial and schematic diagrams.
Communication	<i>Read and interpret</i> mathematical statements or information; <i>explain, display and discuss</i> mathematical information.
Conceptual understanding	<i>Develop understanding</i> of operations and relations for mathematical concepts; <i>identify relationships</i> among different concepts.
Logical reasoning	<i>Explore and link</i> problem elements from logically embedded thought; <i>check a given justification</i> and <i>provide clarification</i> .
Procedural fluency	<i>Perform</i> mathematical operations flexibly, correctly, competently and appropriately.
Strategic thinking	<i>Select or develop</i> a mathematical strategy for a situation arising from a task or context.
Modelling	<i>Interpret mathematical items or information</i> in relation to the situation represented; <i>convert real world problem</i> into a mathematical problem.
Problem solving	<i>Experience the power and usefulness of mathematics</i> in everyday life; <i>apply appropriate skills</i> in solving routine and non-routine problems in a creative way.

General Comments

It is important to note at the outset that the following comments are based on the analysis of a representative sample of scripts and on observations made during the marking exercise. While they inevitably refer to mistakes and misconceptions that can lead to a cumulative impression of poor performance in the question paper, a good number of scripts indicated learners' firm understanding and sometimes excellent capability over the entire syllabus. Where descriptive statistics are cited, it should be understood that they are based solely on the sample of scripts analysed.

A significant proportion of candidates (80.9 %) achieved at least grade 5 in the Mathematics assessment. 13.4 % of the candidates obtained the best possible grade, that is grade 1, and about 19 % of the candidates achieved grade 6, as shown below.



In general, questions that set out to assess learners' knowledge and understanding in the subject were mostly well answered by the large majority of candidates. In particular, many showed competence in carrying out basic arithmetical calculations involving whole numbers. Many pupils

also showed assurance in answering typical questions that required them to recall routine procedures.

However, candidates' responses to **Questions 4, 24, 26, 32, and 36** indicated that quite many developed a shallow understanding of basic concepts such as number operations and fractions. These questions did not require high order thinking skills. Rather, they called for learners to demonstrate their understanding of number relationships, structures and operations.

In general, candidates showed that they could recall facts, properties (of shapes and angles), and concepts such as area and perimeter but struggled to apply this knowledge in given contexts as evidenced in **Questions 37 and 39**. Indeed, candidates had difficulty in making important connections among the seemingly disparate sets of knowledge base they had constructed. Consequently, this hindered their ability to decide on the best approaches to go about answering some questions (**Questions 41, 44 and 45**). A reason that could potentially explain why learners face these difficulties lies in the way they learn mathematics. Teaching and learning of mathematics has often taken a linear and compartmentalised orientation. This has led to a reinforcement of learners' idea that mathematical facts, rules and concepts are quite distinct from each other and even unrelated. It is of vital importance that learning mathematics is geared away from a narrow emphasis on disparate skills towards a focus on deeper understanding of the relationships that exist among concepts to ensure greater success in the future.

Another key area for improvement concerns building learners' confidence in developing their own strategies to solve problems that are unfamiliar to them. The use of more open-ended tasks that elicit learners' thinking and provide them with the opportunities to choose how to solve a complex task on their own, or explore and make general statements, is strongly encouraged. It is by engaging with tasks that learners construct ideas about the nature of mathematics and discover that they have the ability to either 'make sense of' or 'do' mathematics.

It is equally worthwhile to note that a major weakness observed was learners' limited competence in articulating their thinking in a logical manner. Presentation of candidates' work was often messy and difficult to follow. It is felt that this prevented many of them from developing their thinking.

Providing a conducive environment for learners to express (either in writing or verbally) their thought processes when solving a problem in the class may be beneficial in many ways. It not only allows learners to clarify their own understanding but also helps the educator to better grasp what they know, the misconceptions they nurture, and how these might have developed (Resnick, 1988). For this reason, a key message of this report is to make ample room for learners to verbalise their mathematical understanding in class.

Comments on Specific Questions

Knowledge & Comprehension

Question 3

A high proportion of candidates, mostly from the below average ability group, were not able to state the number of sides that a quadrilateral has. Common wrong answers given were 2, 3, 5 and 6.

Question 8

One out of every two candidates was able to represent half past two on the clockface. This suggests that reading and representing time is problematic for quite many. The different responses obtained revealed misconceptions at different levels. Some confused between the hour and minute hands. As a result, the hour hand was quite often seen pointing to six on the clockface and the minute hand showing two. Others did not seem to know how to interpret and/or represent time given in words. Consequently, some drew the hour hand to show two on the clockface while the minute hand was drawn between two and three (half).

Question 10

Candidates did not fare very well in this question which required them to express $\frac{13}{4}$ as a mixed fraction. A multitude of wrong answers were observed with the most common one being $1\frac{3}{4}$.

Question 13

Fewer than half of the candidates were able to recall what a hexagon is. Thus, quite many could not decide on the number of lines of symmetry a 'regular hexagon' has. A significant number of candidates were attracted to option **A** (4). This was the least well answered multiple choice item.

Question 18

This question required candidates to estimate the mass of an adult elephant. It was meant to assess learners' understanding of the size and units of mass. Option **B** (4500 g) was a good distractor. In general, candidates recognised that the mass of an elephant would be numerically big. However, they seemed to be less sure about the appropriate unit that had to be used. A few candidates gave option **C** (45 kg) overlooking the fact that 45 kg could have been their own mass.

Question 21

Here, candidates had to calculate $\frac{4}{5} \times \frac{2}{3}$. A number of candidates seemed to have difficulties recalling the rules for multiplying fractions. Many set out to convert $\frac{4}{5}$ and $\frac{2}{3}$ into $\frac{12}{15}$ and $\frac{10}{15}$ respectively, before they multiplied the numerators and denominators. However, candidates often multiplied the numerators only to end up with the incorrect answer $\frac{120}{15}$. This shows that these candidates confused the rules of addition/subtraction of fractions (where one finds the HCF of the denominators, and converts the fractions into their equivalents with the HCF as a common denominator) with that of multiplying fractions.

Question 23

While it was felt that this question would have allowed candidates to score marks easily, only a relatively small number of candidates were able to answer all the parts correctly. Many identified the missing solid shape as a '*rectangle*' instead of a '*cuboid*', and identified only 2 pyramids in the picture.

Question 25

As mentioned above, candidates did not seem to have understood, let alone master, the rules required to perform arithmetical operations involving fractions correctly. Here, candidates were required to subtract $\frac{3}{5}$ from $1\frac{2}{5}$. It was common for candidates to overlook the whole number '1' in $1\frac{2}{5}$ and subtract 2 from 3 in the numerator, thus ending up with the answer $\frac{1}{5}$. Some even re-wrote $1\frac{2}{5} - \frac{3}{5}$ as $\frac{3}{5} - 1\frac{2}{5}$ and obtained $1\frac{1}{5}$ as answer. It is worth pointing out that correct answers quite often resulted from mathematically incorrect working. For example, it was common to see candidates write:

$$\frac{5}{5} + \frac{2}{5} = \frac{7}{10} - \frac{3}{5} = \frac{4}{5}.$$

Question 29

Performance in **Question 29** was moderately good. It appeared that candidates did not understand the structure of decimal numbers adequately. In general, candidates found it difficult to deal with the place value of the figures in the decimal numbers. 3.8 litres was often represented as 0.38 or 3.08 in the working of candidates. Finding the correct position of the decimal point after subtracting 3.8 from 11.25 was often problematic.

Question 36

This was the least well answered question under the '*Knowledge & Comprehension*' assessment objective. It is important to highlight that candidates did not have to do any calculations in this question. Rather, they had to use the information that was provided and relate it to their knowledge and understanding of number operations to deduce the answers.

36. Tina knows that

$$\boxed{521} \times \boxed{147} = \boxed{76\,587}$$

Without doing any calculation, help Tina to fill in the empty boxes below.

(a) $\boxed{76\,587} \div \boxed{521} = \boxed{\quad\quad\quad}$

(b) $\boxed{52.1} \times \boxed{1.47} = \boxed{\quad\quad\quad}$

(c) $\boxed{\quad\quad\quad} \times \boxed{147} = \boxed{76\,587} + \boxed{147}$

In general, candidates found part (a) relatively straightforward. The majority were able to deduce that 76 587 divided by 521 would give 147.

Part (b) was less well tackled, with many recognising that the answer would be related to the number 76 587, but again, not being able to figure out where the decimal point should be in that number. This indicates that those candidates did not understand that multiplying or dividing a number by factors of 10 effectively moves the decimal point in that number.

Part (c) proved most challenging with very few candidates being able to relate the part question to the information given. The main problem in this question is that most candidates did not actually understand, or failed to recall, what multiplication of one number by another actually means. Many lost precious time by using trial and error unsuccessfully.

Application

Question 4

As straightforward as **Question 4** seemed to be, only a little more than half of the cohort was able to state the fraction that was found exactly between $\frac{4}{7}$ and $\frac{6}{7}$. A significant number of candidates felt they had to 'do something' with the two given fractions. As a result, they either subtracted $\frac{4}{7}$ from $\frac{6}{7}$ or added the two fractions. Thus, $\frac{2}{7}$ and $\frac{10}{7}$ were incorrect answers that occurred frequently.

Question 24

Performance in **Question 24** was rather low. Many did not seem to have read the question carefully enough while some did not understand the requirements of the question. As a result, a very common mistake observed was for candidates to shade 3 additional triangles to represent $\frac{3}{5}$. Quite many also gave $\frac{2}{10}$ as answer given that 2 triangles shown in the diagram were shaded.

It was unfortunate that some found the equivalent of $\frac{3}{5}$ as $\frac{6}{10}$ and, in their haste, went on to shade 6 additional triangles instead of 4, overlooking the fact that two of the triangles were already shaded.

Question 30

About a third of the candidates answered the question correctly. This was a fairly low performance given that questions like **Question 30** are regularly set. Arithmetical mistakes in intermediate working were frequent.

30. **£ 1 = Rs 48**

Ben buys a watch for £ 96 and sells it for Rs 7200.
Calculate the **profit** which he makes, **in rupees**.

Mistakes also arose from candidates' attention being focused on the emboldened word 'profit'. In this way, subtracting 48 from 96 or 7200 was a common reaction from candidates who did not seem to realise that the question dealt with different currencies.

<p>Question 30</p> <p>Method 1:</p> $\begin{aligned}\text{£ } 96 &= \text{Rs } (96 \times 48) \\ &= \text{Rs } 4608\end{aligned}$ $\begin{aligned}\text{Profit} &= \text{Rs } (7200 - 4608) \\ &= \text{Rs } 2592\end{aligned}$	<p>Method 2:</p> $\begin{aligned}\text{Rs } 7200 &= \text{£ } (7200 \div 48) \\ &= \text{£ } 150\end{aligned}$ $\begin{aligned}\text{Profit} &= \text{£ } (150 - 96) \\ &= \text{£ } 54\end{aligned}$ $\begin{aligned}\text{Profit, in rupees} &= \text{Rs } (54 \times 48) \\ &= \text{Rs } 2592\end{aligned}$
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Question 32

Representing numbers on a number line is often used as a pedagogical tool to illustrate strategies for counting whole numbers, fractions and decimals. While it was felt that reading a number line is a skill that would have been well acquired, only about a quarter of the candidates were able to state the value of **X**. The wrong answer, $14\frac{1}{3}$ revealed a key misconception some candidates had about reading number lines (third sub-graduation implied $\frac{1}{3}$). The incorrect answer '17' (resulting from $14 + 3$) was also quite often seen despite the fact that **X** was shown to be between 14 and 15. The equally low performance in part (b) of this question clearly showed that candidates had not been adequately prepared to develop their ability to estimate lengths.

Question 33

The concept of percentage remains abstract for the vast majority of learners. Only a quarter of the candidates answered this question successfully. In general, they were mostly from the above average ability group.

33. 40 children are going to Casela Bird Park on a school outing.
20 % of these children have been to Casela Bird Park **before**.

Calculate the number of children visiting Casela Bird Park for the **first time**.

Language seemed to be a major barrier to average and below average candidates. Not being able to make out what was being asked, they randomly carried operations with the figures given in the question. This explains the lack of logical reasoning in the work presented in the working spaces.

The most common wrong answer obtained was 8 as many set out to calculate $\frac{20}{100}$ of 40 .

<p>Question 33</p> <p>Method 1 :</p> <p>'before' → 20 % 'first time' → 100 % - 20 % = 80 %</p> <p>100 % → 40 children 80 % → $\frac{40 \times 80}{100}$ = 32 children</p> <p>Hence, no. of children visiting the Bird Park for the first time = 32</p>	<p>Method 2 :</p> <p>100 % → 40 children 20 % → $\frac{40 \times 20}{100}$ = 8 children</p> <p>No. of children who visited park before = 8</p> <p>Therefore, No. of children visiting park for the first time = 40 - 8 = 32</p>
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Question 35

Very few candidates were able to answer **Question 35** correctly. This further supports the view that candidates had an inadequate understanding of percentages.

35. The price of a mobile phone is increased by 10 %.
 The **new price** of the mobile phone is Rs 3960.
- Calculate the **original price** of the mobile phone.

As is frequently the case, a common mistake was to calculate 10 % of Rs 3960 (= Rs 396) and to subtract the result obtained from Rs 3960 to get the original price of the mobile phone.

A non-negligible number of candidates felt that 100 % represented Rs 3960. Consequently, they found the original price by calculating $\frac{3960}{100} \times 90 \%$, and obtained Rs 3564.

Question 35

$$\begin{array}{ll} 110 \% \longrightarrow & \text{Rs } 3960 \\ 100 \% \longrightarrow & \frac{\text{Rs } 3960 \times 100 \%}{110} \\ & = \text{Rs } 3600 \end{array}$$

Hence,
Original Price = Rs 3600

Question 40

A fairly small number of candidates answered this question related to speed successfully.

Mistakes mostly arose from misuse of the formula: $\text{speed} = \frac{\text{distance}}{\text{time}}$. Quite many ended up getting the wrong answer due to slips made in intermediate working. A few candidates also had difficulty to subtract time.

Analysis

Question 41

A considerable number of candidates across ability groups displayed inadequate comprehension of this word problem. The question specifically set out to assess candidates' ability to break down and interpret multi-faceted information and data into their component parts.

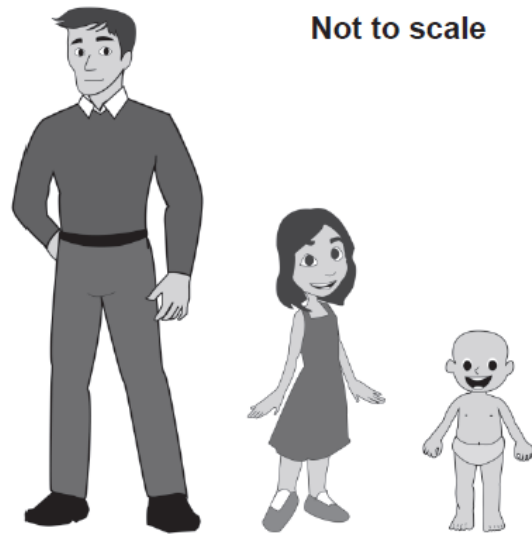
41. The **total** height of Judy and her baby brother is the **same** as the height of their father

Judy is **three times** as tall as her brother.

Their father is 45 cm **taller** than Judy.

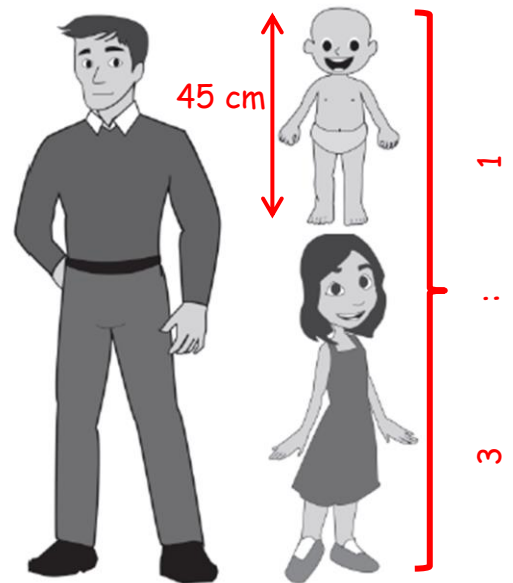
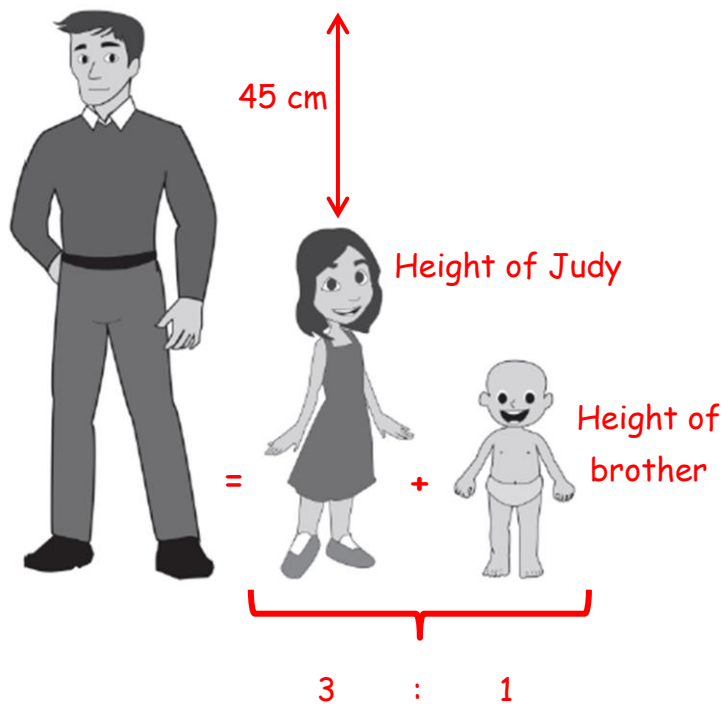
Calculate the height of their father.

Not to scale



Many struggled to translate the information given into meaningful mathematical relations. Often, it seemed that candidates read the information in parts. For example, many rightly concluded that 'three times' implied 'multiply by 3'. However, few were able to recognise whose height had to be multiplied by 3. A very common mistake was to calculate the height of the father by multiplying 45 cm (which many candidates did not realise was the height of the brother) by 3, neglecting the fact that it was Judy who was 3 times as tall as her brother.

Height of father



Height of brother = 45 cm

Implies,

1 share → 45 cm

4 shares → 45×4
= 180 cm

Hence,

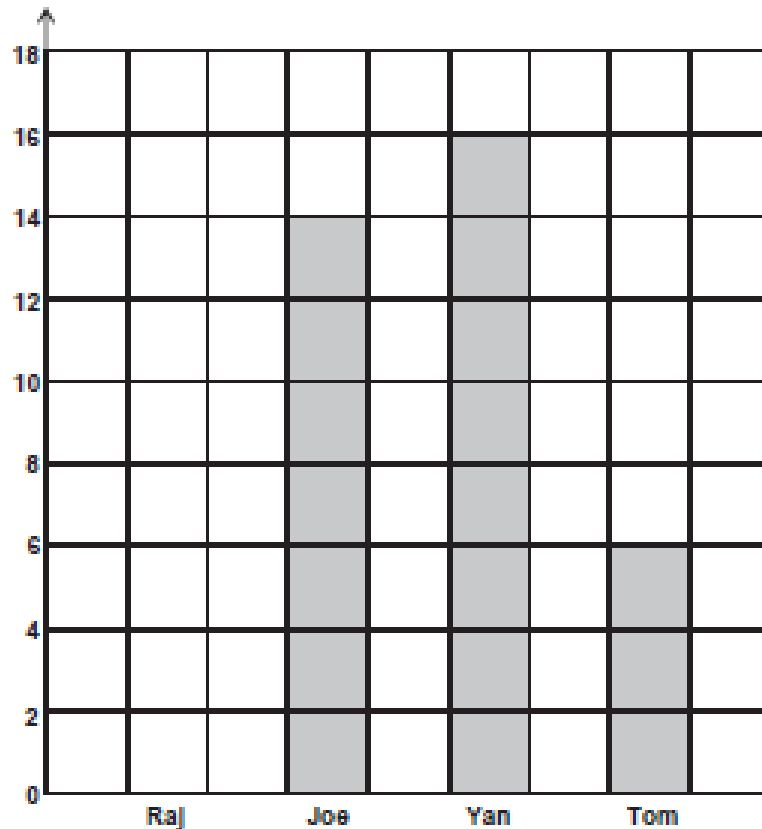
Height of father = 1 m 80 cm

Question 42

This was meant to be an accessible question under 'Analysis' considering that learners are generally good at observing and interpreting bar charts. However, performance here revealed that learners rarely encounter situations where they need to go beyond mere observation and to think more deeply about the type of information presented in a bar chart.

12. Four pupils, Raj, Joe, Yan and Tom solve a problem.
The bar chart below represents the time taken by Joe, Yan and Tom to solve the problem.

Time taken in minutes



- (a) Given that Raj completes the problem twice as fast as Tom, find the time taken by Raj to solve the problem.

Answer: _____ minutes

[

- (b) On the bar chart above, draw the bar representing the time taken by Raj to solve the problem.

[

- (c) Which pupil took the most time to solve the problem?

Answer: _____

[

- (d) Calculate the average time taken by the four pupils to solve the problem.

The vast majority of candidates could not appreciate the fact that Raj solving the problem twice as fast as Tom meant that Raj took less time to solve the problem. They readily accepted that ‘twice’ necessarily implied a multiplication by 2. Consequently, candidates did not fare well in part (a) in general. However, they were able to score partial marks in subsequent parts of the question.

Question 43

This question also assessed candidates’ ability to make sense of complex information.

43. Rina, Lucy and Ann took part in an archery game.
 They scored an **average** of 600 points in the game.
 Rina scored 360 points.
 Lucy scored **half** the number of points which Ann scored.

 How many points did **Ann** score?

A major hurdle for candidates was to interpret the sentence ‘*Lucy scored half the number of points which Ann scored*’. Many mistook the ratio of the number of points scored by Lucy to the number of points scored by Ann to be 2 : 1 instead of 1 : 2. A considerable number of candidates also overlooked the term ‘average’ and took 600 points to be the total number of points scored by all three: Rina, Lucy and Ann. Consequently, 360 was subtracted from 600 instead of subtracting 360 from 1800 (total no. of points = 600×3). It was common therefore to see candidates write

$$600 - 360 = 240; \quad \frac{240}{2} = 120 \text{ points}$$

$$\text{or } 600 \times 3 = 1800; \quad 1800 - 360 = 1440; \quad \frac{1440}{2} = 720 \text{ points}$$

in their working.

Question 44

Question 44 was not an uncommon question. Similar problem solving tasks had been set in the past (**Question 54 (b)** in CPE 2013, **Question 54 (b)** in CPE 2014 and **Question 55 (b)** in 2016). The question was presented slightly differently. It was felt that the use of the diagrams would facilitate candidates’ understanding of the requirement of the question.

44. The rice found in a sack of rice is used to make both small and large packets of rice.

One sack of rice gives

either 4 small packets of rice **and** 2 large packets of rice,

or 8 small packets of rice **and** 1 large packet of rice.

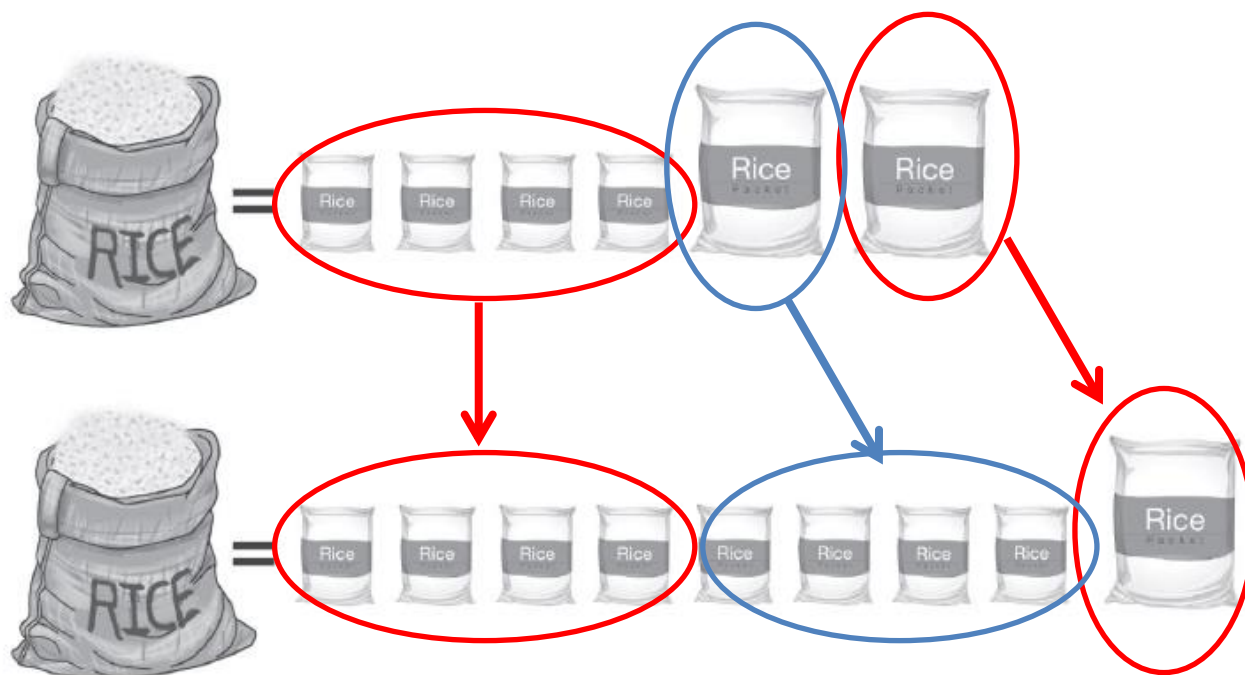
One small packet of rice contains 1.25 kg of rice.

Calculate the mass of rice in one sack of rice.



The challenge here was to recognise that one large packet of rice would weigh as much as 4 small packets of rice which only a few could figure out. Many somehow arbitrarily assumed that one large packet of rice would weigh twice as much as one small packet and based their subsequent calculations on this incorrect assumption. Occasionally, after obtaining the mass of one sack of rice, candidates went on to calculate the mass of the two sacks of rice.

Question 44



From the above diagram,

$$\begin{aligned}\text{mass of 1 large packet of rice} &= \text{mass of 4 small packets of rice} \\ &= 1.25 \text{ kg} \times 4 \\ &= 5 \text{ kg}\end{aligned}$$

Therefore,

$$\begin{aligned}1 \text{ sack of rice} &= 12 \text{ small packets of rice} \\ &= 1.25 \text{ kg} \times 4 \\ &= 15 \text{ kg}\end{aligned}$$

Or,

$$\begin{aligned}1 \text{ sack of rice} &= 3 \text{ large packets of rice} \\ &= 5 \text{ kg} \times 3 \\ &= 15 \text{ kg}\end{aligned}$$

Question 45

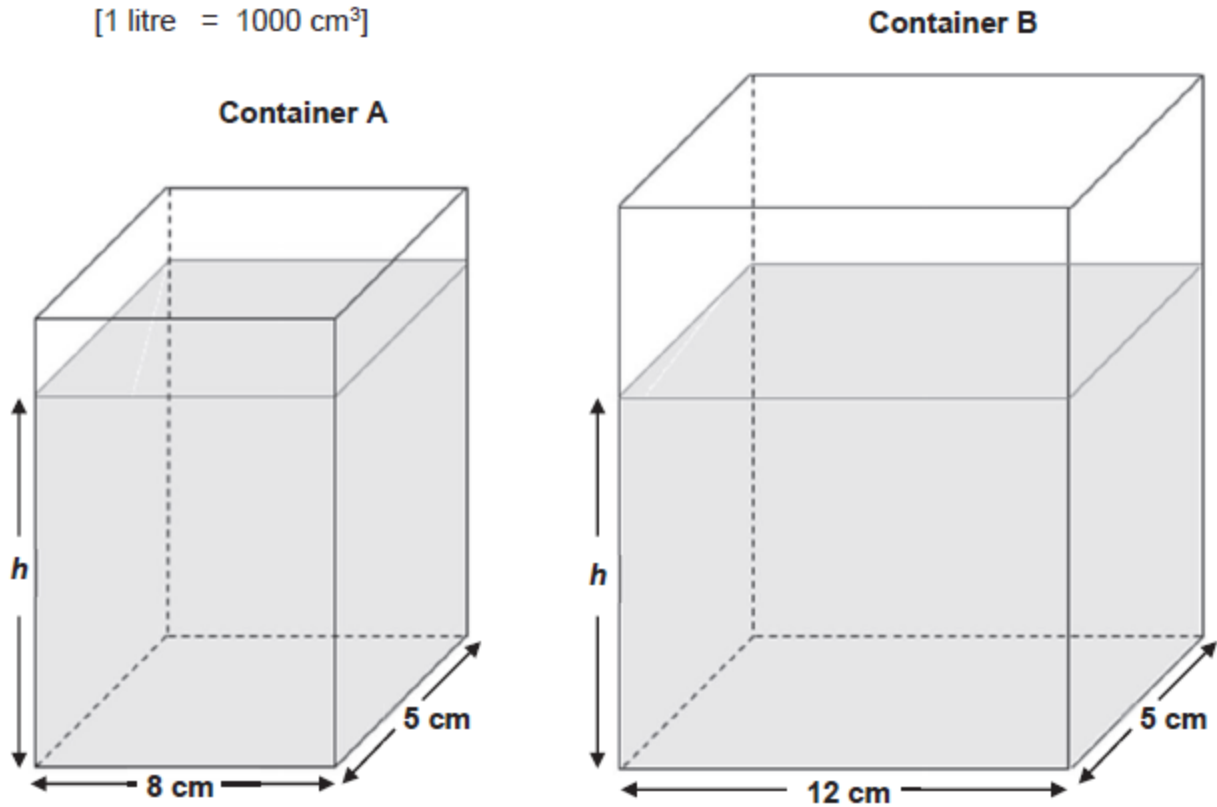
This was by far the least well answered question in the paper which again hints at candidates' lack of confidence to tackle problems that they may not have encountered previously.

45. The diagram below shows two containers, **Container A** and **Container B**.

One litre of water is distributed between the two containers so that the height of water, ***h***, in the containers is the **same**.

Calculate the height of water, ***h***.

[1 litre = 1000 cm³]



There were several approaches that could have been adopted to solve this question. It is felt that a considerable number of candidates could have successfully answered the question had they presented or articulated their thinking process in an organised manner.

Many candidates mistook the phrase '*one litre of water is distributed between the two containers...*' to either mean that one litre of water was equally distributed between the two containers or that each of the containers contained one litre of water. Consequently, they either started off by dividing 1000 cm³ by 2 or by multiplying 1000 cm³ by 2.

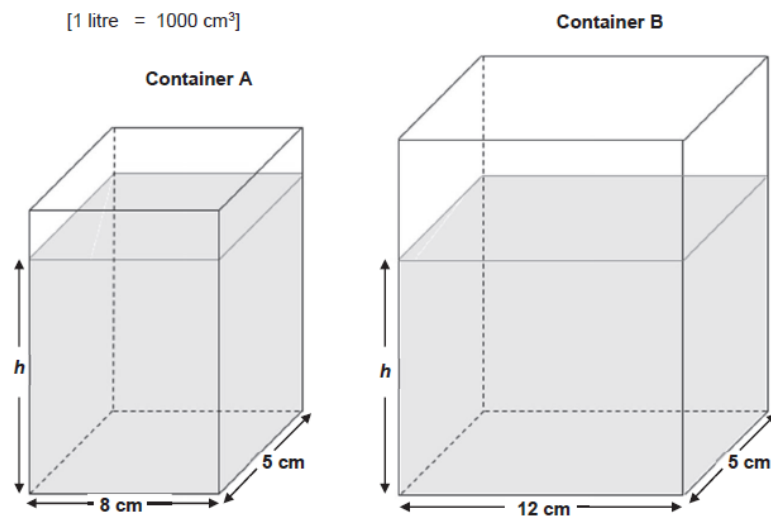
While many reckoned that combining the two containers would have constituted a larger container whose length would have been 20 cm (8 + 12), they omitted the fact that the width of

the combined container would have remained unchanged. They calculated the width to be 10 cm (5 + 5) instead.

The trial and error approach was often used as well although only a few candidates made it successfully to the end.

Question 45

Method 1:



$$\text{Area of cross-section of container A} = 8 \times 5 = 40 \text{ cm}^2$$

$$\text{Area of cross-section of container B} = 12 \times 5 = 60 \text{ cm}^2$$

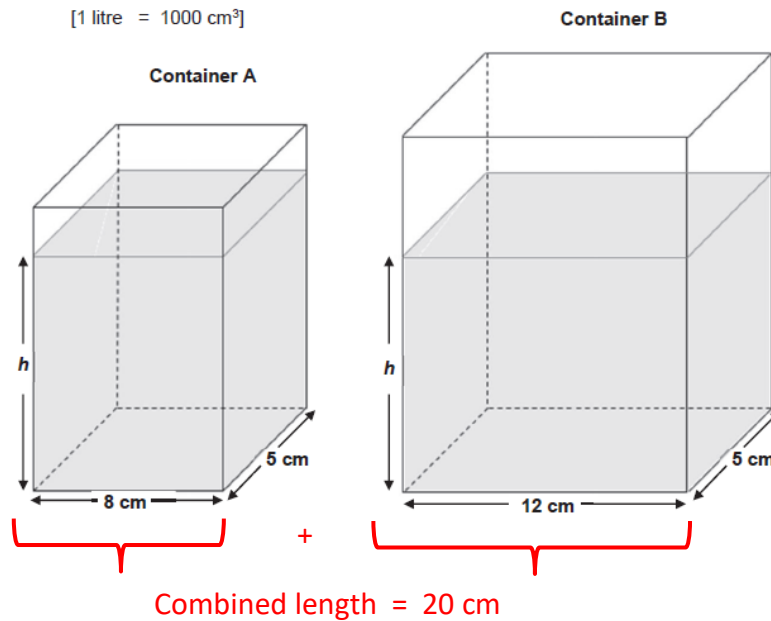
$$\begin{aligned} \text{Total cross sectional area} &= (60 + 40) \text{ cm}^2 \\ &= 100 \text{ cm}^2 \end{aligned}$$

Hence,

$$\begin{aligned} \text{Height of water, } h &= 1000 \text{ cm}^3 \div 100 \text{ cm}^2 \\ &= 10 \text{ cm} \end{aligned}$$

Question 45

Method 2:



$$\begin{aligned}\text{'Combined' cross sectional area} &= 20 \text{ cm} \times 5 \text{ cm} \\ &= 100 \text{ cm}^2\end{aligned}$$

Hence,

$$\begin{aligned}\text{Height of water, } h &= 1000 \text{ cm}^3 \div 100 \text{ cm}^2 \\ &= 10 \text{ cm}\end{aligned}$$

References:

Mauritius Examinations Syndicate (2016), *Annual Programme for the Primary School Achievement Certificate (PSAC) Assessment 2017*, published in the Government Gazette of Mauritius 2016.

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