MATHEMATICS

Paper 9709/12
Paper 12

Key messages

It has been noted that some candidates are omitting essential working. Candidates should be reminded to include all necessary detail in their working to ensure that all possible marks can be gained. Insufficient detailed working was noticeable in **Question 10(iii)** in which the process of finding the difference of the two numbers obtained by substituting the limits needed to be shown. The use of calculator functions for supplying the answers is not sufficient.

General comments

The paper was generally well received by candidates, with many high quality and detailed responses seen. The paper gave all candidates the opportunity to show what they had learned and understood, with some questions that provided more challenge for candidates.

Comments on specific questions

Question 1

This question was an accessible start to the paper with many candidates demonstrating a good knowledge of the binomial expansion. Candidates were very often able to write down the relevant terms, form an equation and solve it correctly. Weaker responses sometimes did not include the term in x in their expansion of $(1+\frac{x}{2})^6$ and therefore only had one term to equate to 3.

Question 2

Most candidates demonstrated a very good understanding of the techniques required. Some candidates obtained the wrong gradient or did not use the midpoint.

Question 3

This question appeared to be more challenging for candidates, with many candidates not identifying the need to integrate or wrongly combining integration with the equation of a straight line. Many candidates equated $\frac{dy}{dx}$ to the gradient of the line joining the two points to find k. Some candidates realised the need to find the two unknowns by using both points given, in an integrated expression.

Question 4

In both parts of this question many candidates correctly used the formulae for the length of an arc and the area of a sector, although some did not use 2θ as the angle. Many candidates, however, did not find the correct length of ***AT*** or ***BT***, often not realising that angles OAT and OBT were $90°$. Some premature approximation for the lengths in **part (ii)** meant that some candidates' final answers were not sufficiently accurate. Working to at least 4 significant figures should guarantee that the final answer is accurate to 3 significant figures.

Question 5

Many fully correct solutions to this question were seen with stronger responses proving the given result in **part (i)** using Pythagoras' Theorem and carrying out the required differentiation and analysis in **part (ii)**. Weaker responses often had errors in the differentiation, particularly with the $\frac{1}{3}\pi$ and the brackets. This sometimes resulted in unsuccessful attempts at the product rule or forming a function of a function. Candidates who expanded the brackets and differentiated each term separately were generally more successful. Those who used the second differential to determine the nature of the stationary value usually obtained full marks. Candidates who attempted to consider the sign of $\frac{dy}{dx}$ either side of the stationary point did not often show sufficient working. A significant number of candidates did not respond to **part (ii)**.

Question 6

This question, particularly **parts (a)** and **(b)(ii)** were challenging for candidates. In **part (a)** common errors included attempting to expand $\tan(2x+1)$, working in degrees, and using $\pi-$ rather than $\pi+$ to find the required values. Candidates should be reminded to give answers in radians to 3 significant figures and not to 1 decimal place as they do for angles in degrees. **Part (b)(i)** was generally well answered with most candidates identifying the need to use $sin^2\theta + cos^2\theta = 1$ to obtain the given answer in the required form. Some candidates omitted **part (ii)** or substituted the end points of the domain. Stronger responses successfully considered the maximum and minimum possible values of $cos^2 x$.

Question 7

Many correct responses were seen to this question. In **part (i)** many candidates were able to interpret the diagram and the information given to obtain the correct vectors. **Part (ii)** was occasionally omitted by candidates or insufficient working was given in order to fully validate the candidates' answers. In **part (iii)** the method for using the scalar product to find the required angle was familiar to candidates, although sometimes candidates' working was not sufficiently detailed or was inaccurate due to rounding prematurely.

Question 8

The arithmetic series in **part (a)** of this question proved to be very accessible to candidates, with many candidates gaining full marks. Some candidates thought that it was a geometric series and some other candidates identified that it was arithmetic but found the distance that would have been run on the 22nd day rather than the 21st. **Part (b)** appeared to be more challenging for candidates, although many fully correct answers were seen in each of the parts. Candidates who equated the third term divided by the second to the second divided by the first generally needed less working and were more successful than those who used the formulae for ar and ar^2 . Some candidates rounded $\frac{2}{3}$ to 0.67 and their subsequent answers were insufficiently accurate. Some candidates incorrectly stated that the fourth term would be $x-7$, whilst other candidates tried to find a sum to infinity even though their *r* value was greater than 1. Candidates who were unable to find the value of *x* in **part (i)** often omitted the last 2 parts.

Question 9

This question appeared to be very accessible to candidates, with many candidates gaining full marks. In **part (i)** the most common approach was to equate the 2 functions and use the discriminant equal to 0 although some candidates made errors in obtaining the required rearrangement or made errors in the subsequent working. Another approach was to equate the gradients and many candidates usually followed this method successfully. Weaker responses sometimes equated the gradient of *f(x)* to 0 or to *g(x)* rather than its gradient.

In **part (ii)** candidates frequently started their response correctly but sometimes made errors in their working or gave *x* as two values rather than an inequality. In **part (iii)** some candidates found gf(*x*) and then tried to find the inverse of this, rather than the inverse of g and then the composite. Some candidates did all their working correctly but then rejected the solution that *x* could be 0. In **part (iv)** most candidates completed the square correctly but then often, did not give the least value of *f(x)*, or gave the *x* value instead, or gave the coordinates of the minimum point.

Question 10

Part (i) of this question was attempted by most candidates and many fully correct answers were seen. Weaker responses sometimes appeared unsure what to do with the 1 in both the differential and the integral and forgetting to multiply or divide by 2 was quite common. **Part (ii)** was appeared to be more challenging with some candidates attempting to find the coordinates of *B* before they had found *A*. Other candidates sometimes either used *x* as 0 in their gradient or equated the gradient to 0. **Part (iii)** was frequently omitted by candidates or the incorrect limits were used; using the coordinates of *B* or the *y* intercept of the curve when integrating the curve with respect to *x* was quite common. Some candidates successfully integrated the function with respect to *y* but this was a significantly more demanding option. Very few candidates were able to obtain the whole area including the triangle.

MATHEMATICS

Paper 9709/22
Paper 22

Key messages

Candidates must ensure that when a question asks for an answer in 'exact form', that their answer is exact. If an exact answer is not required then the appropriate level of accuracy, as stated in the rubric on the front of the examination paper, must be used. Candidates should also ensure that they have answered the questions fully.

General comments

Candidates appeared to have sufficient time to attempt the paper, with sufficient space to answer the questions. It was clear that some candidates were well prepared for the examination.

Comments on specific questions

Question 1

Most candidates attempted algebraic long division with varying amounts of success. Many candidates were able to obtain a partial quotient of $x^2 - 3x$, however errors in the simplification of the terms within the algebraic long division meant that some were unable to obtain the fully correct quotient and remainder.

Question 2

(i) Many correct solutions were obtained with the majority of candidates using the method of squaring each side of the given equation to obtain a 3-term quadratic equation. There were some errors in simplification which to an incorrect quadratic equation and also errors in the solution of the quadratic equation either by factorisation or by use of the formula. Candidates who attempted to find two linear equations very often made sign errors when attempting to deal with the modulus.

(ii) Many correct solutions were seen, with most candidates using their positive answer from **part (i)**.

Question 3

It was essential for candidates to recognise that they needed to write the equation of the straight line in the form $\ln y = \ln k + a\ln x$, recognising that the gradient of the line was equal to a and then making a suitable substitution to find $\ln k$ and hence k. Correct substitution of the given points into the equation $\ln y = \ln k + a\ln x$ to form two simultaneous equations was also acceptable, as was solving simultaneously the equations $e^{3.96} = k \times 0.22^a$ and $e^{2.43} = k \times 1.32^a$. Many candidates were unable to progress much further than finding the gradient of the line. Incorrect use of the coordinates given was a common error in this question. Some solutions showed a correct method to obtain the value of a, but followed with an incorrect substitution to attempt to find the value of $\ln k$ and hence k. Premature approximation in earlier calculations in some cases meant that an inaccurate final value for k was obtained. However, there were candidates who produced completely correct, well set out solutions, showing a good understanding of the straight line theory necessary.

Question 4

(i) Most candidates were able to apply the iterative method required and obtain sufficient iterations to the correct accuracy and hence the final answer to the correct accuracy. Common errors included not giving enough iterations to justify the final answer and not giving the final answer to the correct accuracy required. Some candidates misread the question and used the iterative formula $x_{n+1} = \dfrac{1}{\ln(2x_n)}$ rather than the correct $x_{n+1} = \dfrac{x_n}{\ln(2x_n)}$. Candidates must ensure that they read the question carefully.

(ii) Very few correct solutions were seen with many candidates using their answer to **part (i)** to answer this part. The word 'exact' was used in the question, which should have highlighted this error as their answer to **part (i)** was not exact. Some candidates stated a correct equation but did not identify the word 'exact' and gave a decimal equivalent of $\dfrac{\mathrm{e}}{2}$.

Question 5

This question was intended that candidates make use of the product rule, equate their result to zero and solve to find x and then y. Use of the quotient rule was also acceptable, so long as a correct rewriting of the exponential term had taken place. Many candidates attempted to use the product rule but were unable to differentiate $\mathrm{e}^{-\frac{1}{2}x}$ correctly. When candidates attempted to equate their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to zero, several candidates were unable to do this correctly, not realising that $\mathrm{e}^{-\frac{1}{2}x} \neq 0$ and thus attempting to deal with the resulting linear factor in x. There were candidates who, having obtained a correct value for x, did not give an exact value for y.

Question 6

(a) With a given answer to work towards, most candidates attempted to work with a multiple of $\ln x$, or equivalent, from integration. There were many correct integrals of $\dfrac{3}{2}\ln x$ or equivalent. The question demanded that a given result be shown. It was therefore essential that each step of the application of the limits, use of the subtraction rule for logarithms and use of the power rule for logarithms (in either order) be shown. Many candidates did not show the necessary detail needed in a question of this type. Several candidates also used their calculators to attempt to show their decimal result was equal to $\ln 27$.

(b) Candidates needed to identify that a double angle formula was needed to rewrite the integrand. Candidates who made use of the correct double angle formula, often gave an exact answer as required. Several candidates did not successfully deal with integrands involving the square of a trigonometric expression.

Question 7

(i) Many candidates identified the correct approach in this question. There were errors in the coefficient obtained from the parametric differentiation of the given equations and also errors in simplifying these results when applied to $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}\theta} \times \dfrac{\mathrm{d}\theta}{\mathrm{d}x}$. Errors also occurred in the simplification of this result when the substitution of $\theta = \dfrac{\pi}{6}$ was attempted.

(ii) Many candidates carried forward their errors in simplification from **part (i)** into **part (ii)**, which made it difficult to progress. Some correct solutions were seen, with the occasional use of degrees rather than radians used in the final answer. Candidates should always ensure that they are working with the correct measure of angles when attempting questions of this type.

Question 8

(i) Most candidates were able to gain marks in this part. It should be noted that the value of the angle was required to 2 decimal places, meaning that several candidates did not gain the final accuracy mark as an answer of $67.4°$ was given. Some candidates made sign errors when attempting to find the value of α. It was also expected for the value of *R* to be evaluated either as 1.3 or an equivalent fraction. A result of $\sqrt{1.69}$ was deemed to be insufficiently simplified.

(ii) Provided the result from **part (i)** was used correctly, many candidates were able to obtain at least one of the solutions. Many candidates had difficulty in dealing with the fact that if $\theta + 67.38° = 52.02°$ a negative angle was obtained and that they should be considering $\theta + 67.38° = 52.02° + 360°$. It was important that candidates appreciate the meaning of the word 'Hence' and use their results from **part (i)**. Several candidates also stated the incorrect $\cos(\theta + 67.38°) = \cos\theta + \cos 67.38°$.

(iii) Very few correct responses were seen to this part. It was intended that the expression from **part (i)** be used so that the given expression could be written as $\left(3 - 2(1.3)\cos(\theta + 67.38°)\right)^2$. Some candidates progressed this far, but did not deduce that they could make use of the maximum value of $\cos(\theta + 67.38°)$ being 1 and that the minimum value of $\cos(\theta + 67.38°)$ is -1. A simple substitution then yields the correct results. Some candidates expanded out the brackets and not progress much further. Candidates should be guided by the mark allocation and in this case, the amount of space given as to the amount of work that is necessary to complete the question.

MATHEMATICS

Paper 9709/32
Paper 32

Key messages

Candidates must ensure that they show clear and detailed working, particularly when a question asks to obtain a given answer. Candidates must ensure they write clearly and do not overwrite one solution with another as this can often make their response illegible. Candidates must ensure that when a question asks for an answer in 'exact form', that their answer is exact. If an exact answer is not required then the appropriate level of accuracy, as stated in the rubric on the front of the examination paper, must be used.

General comments

The paper appeared fairly challenging for many candidates with **Questions 1** on logarithms and indices, **Question 3** on factors and remainders, **Question 5** implicit differentiation and **Question 7** on complex numbers, were not well answered by many candidates.

It was noted that several candidates did initial work in pencil and then over-wrote in pen, this can result in work being illegible. It is important to remind candidates not to do this, and that it is preferable to put a line through work and re-write it if corrections are necessary.

A number of simple errors in basic algebra or arithmetic were seen in candidates' responses throughout the paper. Attention to detail is an essential part of good mathematical work. Among the errors seen, not using brackets appropriately is the most common, by omitting these, this can often result in incorrect further work and loss of accuracy.

Candidates commonly did not give answers to the required degree of accuracy or in the required form and often did not include sufficient detail of working which was required.

Comments on specific questions

Question 1

Most candidates presented confident responses to this question, however, many started with the incorrect statement $5\ln\left(4-3^x\right)=5\ln 4-5\ln 3^x$. A large proportion of the candidates who started with the correct statement $4-3^x=e^{1.2}$ made no further progress because they then continued to take logarithms term by term afterwards. There were several errors noted in calculator work; the correct expression $\dfrac{\ln\left(4-e^{1.2}\right)}{\ln 3}$ did not always lead to a correct answer. Some candidates appeared to show lack of confidence in manipulating logarithms, which was reflected in the commonly seen, incorrect step $\dfrac{\ln A}{\ln B}=\dfrac{A}{B}$. Several candidates did however identify that using logarithms in base 3 was an efficient way to finish.

Question 2

The majority of candidates started with a correct application of the quotient rule. There were some errors in misquoting the rule, and some candidates omitted the denominator altogether. Candidates who made their working clear by setting out their *u* and *v* and quoting the correct rule usually scored the first M mark. A few candidates used the product rule, but the derivative of $(1-x^2)^{-1}$ proved challenging.

Most candidates recognised the need to equate the derivative to zero, but subsequent errors in algebraic manipulation led to many a sign error and an incorrect quadratic. The majority of candidates did reject the inadmissible solution at the end.

Question 3

This question appeared to be particularly challenging for candidates. Most candidates started by attempting to divide x^4+3x^3+ax+b by x^2+x+1. The first step of this division was often correct, but subsequent errors in the algebra and arithmetic, including errors caused by not allowing for the absence of an x^2 term in the quartic, led to many incorrect quotients and remainders. Some difficulties also arose when dealing with the signs of terms involving *a* and *b* : $b+1$ in place of $b-1$ in the remainder was common, despite having everything correct up to that point. Dealing with variations of $(a+2)x+x$ in the remainder also caused some confusion with x appearing and disappearing. Having reached a linear remainder, many candidates did not go on to set their remainder identically equal to $2x+3$, and to solve for *a* and *b* accordingly.

Some candidates opted to use the remainder theorem, but the majority of these used decimal values for the roots of x^2+x+1 rather than work through with exact surd values. Other candidates substituted the value for the root in the quartic, but not in the remainder.

A few candidates opted for the variation of subtracting $2x+3$ from $p(x)$ before doing the long division and setting their remainder equal to 0. A common error was for candidates to treat $2x+3$ as a factor.

Question 4

(i) There were many fully correct solutions to this part of the question. Almost all candidates obtained $R=\sqrt{7}$ and the majority used a correct method to find α, although many gave this to only 1 or 2 decimal places. The most successful students expanded using the compound angle formula and showed full working. Other candidates recited a formula for carrying out the procedure, although effective in some cases, the error $\tan\alpha=\sqrt{6}$ was common. Some candidates started with the incorrect statements $\sin\alpha=1$ and $\cos\alpha=\sqrt{6}$.

(ii) Most candidates found this question challenging. Many candidates did not work with enough accuracy in answering this question, given that **part (i)** required an answer to 3 decimal places, candidates should have identified that at least 3 decimal places was needed for this part. A common incorrect answer was 13.5. Some candidates attempted to acknowledge the connection between the two parts but solved for 2θ rather than for θ or tried to solve $\sqrt{7}\sin(2\theta+44.416)=2$.

A few candidates disregarded their work from **part (i)** and used alternative methods, usually involving double-angle identities, however these rarely resulted in a correct solvable equation in a single trigonometric function.

Question 5

The majority of candidates started by using implicit differentiation to differentiate the left hand side of the equation. A lot of correct work was seen, but also several sign errors due to omission of brackets when differentiating the second term were noted. Very few candidates did not obtain zero as a result of differentiating the right hand side; several continued with a^3, and a few had $3a^2$.

The phrase 'the tangent is parallel to the *x*-axis' was not understood by all candidates; it was often interpreted as $x=0$ or $y=0$ or the denominator of the expression for the derivative is zero or, occasionally, $\frac{dy}{dx}=1$. Those candidates who correctly set the derivative equal to zero usually obtained $y=4x$ but they often overlooked the possibility of $y=0$. Very few candidates earned the mark for mentioning and rejecting $y=0$, instead a few selected this and rejected $y=4x$. Several candidates did not progress further than $y=4x$. Those who did refer back to the original equation to find *y* often made errors in dealing with the cubes and the error $(4x)^2=4x^2$ was common.

Question 6

The majority of candidates recognised that the correct first step was to separate the variables, and most gained the first two marks by getting as far as $\ln(x+2)$. Most candidates went on to obtain $k\ln\left(\sin\frac{1}{2}\theta\right)$ but only a minority had $k=2$. Many candidates found difficulty in the correct simplification and removal of the logarithms. Several candidates obtained a correct expression in terms of $\sin\frac{1}{2}\theta$ or $\cos\frac{1}{2}\theta$, but did not progress further than this. Very few fully correct solutions were seen as the majority of candidates did not attempt the final step to express the answer in terms of $\cos\theta$.

Question 7

(a) Many candidates found this question challenging, often candidates would confuse their working involving *z* or *z** rather than *x* and *y*. Candidates who started by multiplying up by *z** or $x - \mathrm{i}y$ usually reached a correct horizontal equation. Those who tried dealing with the $\frac{\mathrm{i}z}{z^*}$ by turning it into $\frac{iz^2}{z^*z}$ often made arithmetic and algebra errors in their working. Only a minority of candidates who reached a horizontal equation in *x* and *y* went on to consider the real and imaginary parts.

(b) (i) Several candidates did not provide a response to this question part. Of those who responded, several candidates demonstrated an understanding of the equations of basic loci. Some candidates did not draw a recognisable circle, however several circles with the correct radius and centre were seen. Many candidates found drawing the line $\mathrm{Im}\,z=3$ challenging, with several candidates drawing a second circle.

(ii) Many of the stronger responses identified *P* and obtained the correct answer quite easily, with some adding relevant details to their diagram in **part (i)** by way of explanation. Many candidates used their inaccurate diagrams from **part (i)**, with several candidates tried taking measurements from their diagrams, so 1.7, and 1.8 often appeared in the working.

Question 8

(i) This question was well received by the majority of candidates. Most candidates set up an appropriate form of partial fractions, though a minority started with $\frac{B}{x^2+2}$ in place of $\frac{Bx+C}{x^2+2}$. In this question many candidates did not include brackets, which was a common error seen. There were also several cases of miscopying $2x-1$ as $2x+1$. After fully correct working, several candidates misstated their conclusion as $\frac{4}{2x-1}-\frac{1}{x^2+2}$, this however did not affect the marks gained in this part.

(ii) Most candidates integrated their $\frac{k}{2x-1}$ to get a term in $\ln(2x-1)$, but an error in the multiplying constant was common. For candidates who found $C=0$ in **part (i)** the second term was often integrated correctly, although it was common to see an erroneous extra *x* in the integral. Several candidates started from $\frac{-1}{x^2+2}$ instead of $\frac{-x}{x^2+2}$ and gained no credit for an apparently 'correct' integral as it came from wrong working. When *C* was found to be non-zero, candidates then needed to split this term into 2 parts.

Those candidates who had the correct forms of both integrals usually substituted limits correctly, but some made errors in combining logarithms and some candidates did not gain the final mark by not putting the answer into the required format.

Question 9

(i) The majority of candidates attempted to use integration by parts. Most candidates clearly understood the basic process, but there were many errors in the coefficients. It was common to see $\frac{1}{3}$ and $\frac{1}{9}$ in place of 3 and 9, and several candidates did not include the 3 at all. Those who did complete the integration correctly usually went on to use the limits correctly and reach the given result.

(ii) Many candidates appeared to be unsure of whether they were required to look for solutions of $\mathrm{f}(a) = a$ or look for solutions of $\mathrm{f}(a) = 0$. Very few of those candidates who evaluated $\dfrac{4 - 3\cos\frac{a}{3}}{\sin\frac{a}{3}}$ at 2.5 and 3, went on to make the relevant comparisons with 2.5 and 3; many candidates seemingly adjusted their answers to demonstrate a sign change.

Several candidates evaluated an expression such as $\mathrm{f}(a) = a\sin\frac{1}{3}a + 3\cos\frac{1}{3}a - 4$ for a = 2.5 and a =3 and they often completed the argument correctly. Some candidates simply stated '> 0' and '< 0' without giving values, this was not sufficient detail to gain the mark.

Some candidates appeared to be working in degrees, rather than radians as stated in the question stem. There were also some candidates that did not use an appropriate formula, in some cases they often gained or omitted a 3.

(iii) Some candidates appeared to be working in degrees in answering this question, however many candidates answered this question well. Some candidates did not work to the accuracy requested in the question, and some did not give a final conclusion even though they worked through the iteration correctly.

Question 10

(i) The majority of candidates showed a good understanding of the method required, with errors in signs and algebra being a common occurrence. Several candidates started with a correct expression in component form, but then set the scalar product with $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ equal to zero rather than 5. A small number of candidates tried to remove the fractions in the final answer and multiplied their position vector by 3, thus not gaining the final mark.

(ii) The majority of candidates used the correct method to find the angle between the line and the vector perpendicular to the plane. However, many did not give an acute angle as their final answer, and many did not give the angle between the line and the plane. Answers such as $50°$, $130°$ and $-40°$ were common.

(iii) Correct application of the vector product resulted in concise, correct solutions from many candidates. This was by far the most popular and successful method, although sign errors were seen in some solutions. Candidates who used the alternative method of stating two scalar product equations tended to find it relatively difficult to complete the method. Common incorrect methods usually involved confusion between directions and position vectors.

MATHEMATICS

Paper 9709/42
Paper 42

Key messages

Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Cases where this was commonly not adhered to were seen in **Question 3**, **Question 4**, **Question 6** and **Question 7**. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation or a work/energy equation. Such a diagram would have been particularly useful for **Question 3**, **Question 6** and **Question 7**.

In questions such as **Question 1**, where velocity is given as a cubic function of time, it is important to identify that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The paper was generally very well answered by many candidates, although a wide range of marks was seen.

Most candidates work was well presented, but candidates must be reminded of writing their answers clearly in either a black or dark blue pen.

In **Question 6**, the angle θ was given exactly as $\cos\theta = \frac{24}{25}$. In cases such as this, candidates should be reminded that they do not need to evaluate the angle, as this type of problem can often lead to exact answers and so any approximation of the angle can lead to a loss of accuracy.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst still differentiating well between the stronger candidates. **Questions 1** and **2** were found to be the most accessible whilst **Questions 4** and **5** proved to be the most challenging.

Comments on specific questions

Question 1

In this question the displacement is given as a cubic function of time. The constant acceleration equations cannot be used here. Since the velocity is required it is first necessary to differentiate s to find the velocity, v. It also necessary to differentiate v in order to find the acceleration, a. In order to answer the question, the expression for a must be set equal to zero and then solved for t. This value of t is now used in the expression for v. Most candidates made a very good attempt at this question but some who found the value to be negative gave their answer as a positive. Since the question asked for velocity and not speed, the negative is the correct answer.

Question 2

(i) Most candidates answered this question correctly. The acceleration is given between t = 30 and t = 35 and one method is to equate the given acceleration to the gradient of the line between those two points to find V. This was the technique used by the majority of candidates.

(ii) Since the total distance travelled is given, the most common method used was to evaluate the area under the velocity-time graph. This can be broken down in a number of different ways involving triangles, rectangles and trapezia. One of these regions can only be expressed in terms of U. When the sum of the areas is equated to 375, this leads to an equation in U. Although most candidates answered this question correctly, the most common errors seen involved the numerical calculations of the areas.

Question 3

This is a question where the other angles in the right angled triangle are not given directly. However it should be clear that is the 3, 4, 5 triangle such that sin PAB = 0.8 which means that there is no need to evaluate the angles in the triangle. Despite this most candidates found the angles as 36.9 and 53.1 but there is a danger that this will lead to a lack of accuracy. There are many different possible approaches to this problem. The most straightforward method is to resolve forces vertically and horizontally. This leads to two equations relating the tensions T_A and T_B in the two strings. The solutions of these simultaneous equations will produce the required answer. A common error was to disregard the weight of the particle. Other common errors seen were to either wrongly assume that the tensions in the two strings were the same, or to assume that the other two angles in the triangle were both 45°.

Question 4

(i) In this question it is necessary to use the given information in the relationship $P = Fv$ to determine the power required to maintain a constant given speed. Since the lorry is travelling at a constant speed, the resistance force acting on the lorry must exactly balance the driving force. hence the driving force is 3000 N. Most candidates found this correctly and used it to find the required power.

(ii) This question specifically asks for the problem to be solved using an energy method and full marks could not be gain unless this advice was followed. Using the given information, it is possible to determine the loss in kinetic energy and the gain in potential energy as the lorry moves from A to B. Since the question asks for the height of B above A, either this value or the distance travelled up the plane must be used in the equation. The energy equation will then take the form
KE loss = PE gain + WD against resistance. The only unknown in this equation will be the height of B above A which will lead to the required result. Many candidates did not multiply the resistance force of 3000 N by the distance moved up the plane in order to find the work done against resistance. Another error seen was to have incorrect signs in the energy equation.

Question 5

(i) This question was found by most candidates to be the most difficult on the paper. It requires some thought as to how to approach it and again there are several different methods of approach. Using the constant acceleration equations for the motion of A, the distance it has travelled t seconds after it was projected is $20t - \frac{1}{2}gt^2$. When considering particle B, t seconds after A is released particle B has travelled a distance $\frac{1}{2}g(t-1)^2$. Many candidates did not take into consideration the time lag between the times of projection. Once these two expressions have been found they will then collide when the sum of the two distances equals 40 m. However, many candidates thought that they would meet when these two distances were equal. An alternative approach is to find the distance travelled by A one second after projection and to find its velocity. The motion of the two particles can now be modelled using the same t value. Once the time at which collision occurs is found then the height at this time can be determined.

(ii) In this part of the question the time at which the collision occurred can be used in an equation of the form $v = u + at$ for each particle. It must be remembered that the times used will differ by one second. Candidates who did not find the correct solution in **part (i)** were still able to earn the method marks in this part. The most common error was to use the same t value for both particles.

Question 6

(i) In this question it is given that a constant force is applied to the block and so the constant acceleration equations may be used. From the given information we have, $u = 0$, $s = 4.5$ and $t = 5$ and using the equation $s = ut + \frac{1}{2}at^2$ the acceleration can be determined. Once the acceleration is known then Newton's second law of motion can be applied. The forces acting on the block are a component of the 6 N force and the opposing friction force. This equation will give the required value of the friction force. Most candidates successfully found the acceleration but often used the 6 N force rather than a component of it when applying Newton's second law.

(ii) In this part of the question the value of μ is given correct to 3 significant figures and so care must be taken to show all working. As the friction force is known, all that is now required is to find the normal reaction, so that the equation $F = \mu R$ can be used to find μ. When resolving vertically there are three forces involved, the weight, a component of the 6 N force and the normal reaction and hence the normal reaction, R, is a combination of the weight and a component of 6 N. Many candidates incorrectly stated that $R = 3g$ and were then unable to achieve the given result. Another common error was a misreading of the question and because an angle was given, several candidates assumed that the motion took place on an inclined plane rather than on the horizontal as stated.

(iii) Many candidates followed through the incorrect friction force from **6(i).** In this part the 6 N force had now been removed and so $R = 3g$ is correct and so the friction force is now $F = 0.165 \times 3g$. This is the only force now acting on the block and leads to an acceleration of $0.165g$ and this can be used in the constant acceleration formula $v = u + at$ and the time taken to come to rest can be found. Many candidates correctly obtained this value but in the question it asks for the total time in motion which requires the extra 5 seconds to be added.

Question 7

(i) This question involves a system of connected particles. There are three possible equations of motion and any two of them will enable the problem to be solved. Newton's second law can be applied either to particle P or to particle Q or to the system of both particles. The system equation will not involve the tension in the string. Solving any two of these three equations will give the tension and the magnitude of the acceleration. A common error made by candidates was to assume that the force acting on P was $0.3g$ rather than a component of the weight down the plane. Most candidates made a very good attempt at this part of the question.

(ii) The constant acceleration equation $s = ut + \frac{1}{2}at^2$ can be used here to find the required time since $u = 0$, $s = 0.8$ and $a = 0.4$ as found in **part (i)**. A common error was for candidate to use $a = g$ in this equation.

(iii) Once the string becomes slack particle P will continue to move up the plane until it comes to rest and will then return to the original position when the string will again become taut. There are various methods which could be used. One method is to determine the time taken before P comes to rest and then the total time will be found by doubling this. The speed of P as the string becomes slack is simply $v = at = 0.8$ The acceleration of P while it is moving up the plane will be a component of g namely $-g \sin \theta = -6$. This can be used in the equation $v = u + at$ to find t. An alternative is to use the equation $s = ut + \frac{1}{2}at^2$ with $s = 0$, $u = 0.8$ and $a = -6$ and this will give the total time that the string is slack. As the question asks for the time from the instant that P is released and so it must be remembered to add the 2 seconds found in **part (ii)**.

MATHEMATICS

Paper 9709/52
Paper 52

Key points

Candidates should be reminded to give answers to 3 significant figures unless otherwise stated in the question. As stated in the rubric on the front cover, $g = 10$ should be used not 9.8 or 9.81.

If $\sin\theta = \frac{3}{5}$ is given in the question then candidates should recognise that $\cos\theta = \frac{4}{5}$ and $\tan\theta = \frac{3}{4}$ and use these rather than find the value for θ, as this can result in a loss of accuracy.

General comments

Most candidates work was neat and well presented. Candidates should always refer to the formula booklet provided if in doubt of a specific formula.

Candidates should be reminded that an answer should be given to 3 significant figures unless otherwise stated in the question. This means that they should work to at least 4 significant figures.

The questions that candidates found more accessible were **1**, **2(i)**, **2(ii)**, **6(i)** and **6(ii)**, whilst the questions found more challenging were **5(ii)**, **6(iii)** and **7(iii)**.

Comments on specific questions

Question 1

This question was quite well answered by many candidates. Some candidates tried to use $T = \frac{\lambda x}{L}$, which was an incorrect method. In order to answer the question correctly, the most successful approach was to use an energy method.

Question 2

Both parts of this question were generally well answered.

Question 3

Many candidates were successful in answering this question. The most efficient method for answering this question was to use Newton's Second Law vertically and then apply a 4-term energy equation.

Question 4

(i) This part of the question was generally well answered. Candidates needed to use $\tan 30° = \frac{v_y}{v_x}$, rather than $\tan 30° = \frac{y}{x}$, and also $v_y = 30 - V\sin 60°$ should have been used rather than $v_y = V\sin 60° - 30$.

(ii) This part was generally well answered. Candidates were required to find x and y after 3 seconds. Pythagoras' theorem then resulted in the required distance.

Question 5

(i) To answer this part the first step was for candidates to find the radius of the circle which was 0.5 sin 30 = 0.25 m. The next step was to resolve vertically in order to find the tension. Once the tension was found, Newton's Second Law horizontally gave the required velocity. Many candidates provided good answers to this part of the question.

(ii) To solve this part of the question candidates were required to resolve vertically and also to use Newton's Second Law horizontally. By solving the two equations, the two tensions could be found. Many candidates appeared to find this part of the question more challenging.

Question 6

(i) Most candidates answered this part of the question correctly.

(ii) A small number of candidates did not solve the equation $0.45x^{\frac{1}{2}} - 1.5 = 0$. Overall, this part of the question was well answered.

(iii) Many candidates found this part of the question challenging. The first step was to integrate the equation from **part (i)**. This often resulted in v =... and not $\frac{v^2}{2} = \ldots$. For candidates who acquired the correct integration it was then necessary to substitute x = 11.1, the value found in **part (ii)**, this gave the constant of integration, c, to be $\frac{50}{9}$. Then by substituting $x = 0, \frac{v^2}{2} > \frac{50}{9}$ to result in $v > \frac{10}{3}$.

Question 7

(i) This part of the question was generally well answered. To be successful in answering this question candidates were required to take moments about the line AD.

(ii) Many candidates correctly answered this part of the question.

(iii) This part appeared to be a challenging question for candidates. Candidates needed to recognise that AD would make an angle of 40° or 20° with the vertical. Candidates then needed to take moments about A for both situations. This resulted in $W \times AG \sin 10° = 7 \times 2.4 \cos 40°$ and $W \times AG \sin 10° = 7 \times 2.4 \cos 20°$.

MATHEMATICS

Paper 9709/62
Paper 62

Key messages

Candidates should be encouraged to use simple diagrams as to aid understanding in many questions by helping to visualise the conditions stated within the question.

It is essential to work to at least 4 significant figures throughout a question to ensure that the required accuracy can be achieved. Many solutions did not achieve the required accuracy of the questions as many candidates had worked to 3 significant figures or 3 decimal places earlier in the process.

Communication of processes by showing workings assists both the candidate to check their work and allows for method marks to be awarded credit when there are numerical errors in calculations.

General comments

Candidates appeared to have had sufficient time to complete the paper, although some candidates would appear to have spent a significant amount of time on **Question 3**. The use of a ruler is expected when drawing statistical diagrams such as a histogram.

Questions 4, 5 and 6 were generally answered well, while **Question 2 and 7** appear to have been more challenging for candidates.

It was noted that several candidates appeared to have made errors within their calculations because of misreading their own writing. Candidates should ensure that their presentation is clear such that they can check their work and that marks can be awarded appropriately.

Comments on specific questions

Question 1

(i) Many candidates were successful in attempting this question. Good solutions included reordering the data to facilitate identification of the median and quartiles. Most solutions correctly stated the value for the median, although some adjusted their value to coincide with an original data value. A significant number of solutions did not identify the quartiles accurately by not finding the mid-values of the data above or below the median. A number of solutions were also adjusted to coincide with original data values. However, the interquartile range was attempted by almost all candidates. Weaker solutions did not reorder the data initially and found the mid-value and quarter-values of the question data.

(ii) Good solutions clearly stated that 110 was significantly different from the other data values and would make the mean unsuitable. Weaker solutions stated that extreme values affect the mean, however did not link this to the question context, therefore candidates could not gain credit.

Question 2

(i) Although there was no requirement to construct a tree diagram, it is a significant aid to understanding the context of the question and better solutions often started with one. Almost all candidates made some progress towards calculating the value of *x* but did not always included sufficient method for a 'show' question, where a simple two term equation was necessary to justify the given answer.

(ii) Better solutions recognised that the conditional probability calculation could be based upon the information provided in **part (i)**. A small number of good candidates recalculated the probabilities using the standard textbook approach. Weaker responses used the complements of the required probabilities or did not calculate the probabilities for all the necessary outcomes required for the denominator and so gained little credit.

Question 3

(i) Many candidates found this question challenging. Good solutions made effective use of the space around the data table to calculate both the class width and the frequency density. Since speed is continuous, the upper/lower boundaries of the classes needed to be used so the best solutions used the main grid-lines on the horizontal axis for values 9·5, 19·5, 29·5 etc. rather than 10, 20, 30 etc. This was acceptable and enabled more accurate drawing of the column lines. Good solutions also showed the axes fully labelled with 'frequency density' and 'speed, km/h'. A common misconception was that the class width was the difference between the table values. A small number of candidates calculated the frequency density as 'class width/frequency'. A large proportion of candidates' graphs included correctly labelled axes, rather than simply stating either 'speed' or 'km/h'. It would be advised for candidates to choose scales that enable graphs to be drawn accurately. A number of speed scales, e.g. 3 cm: 20 km/h or 1 cm: 20 km/h, made it difficult for candidates to achieve the expected accuracy. Weaker solutions showed a bar chart of the data.

(ii) Many good solutions were seen to this question part. Stronger responses constructed a table to manage their calculation, stating the frequency, mid-value and their product before the final calculation. The most common approach was to produce a single calculation; inaccuracy in calculations were often noted here. A common error was to use either the class width or upper boundary in place of the midpoint. Several candidates summed their frequencies inaccurately however did not compare with the information within the question.

Question 4

(i) Almost all candidates used the Binomial distribution accurately. The most frequent error was to omit '8 households satisfied' when interpreting 'at least 8 are satisfied'. Candidates should be encouraged to develop an accurate understanding of the terminology used in this style of question. Many solutions did not gain full credit due to premature approximation. Candidates should be reminded that at least 4 significant figures are expected to be used in all work leading to the final answer.

(ii) Many good solutions were seen using the Normal distribution as an approximation to the Binomial distribution. Better solutions stated the calculations required for the mean and variance before evaluating, identified that a continuity correction was required because the data is discrete, and interpreted that 'more than 84' did not include 84 within the data. A simple sketch of the normal curve was often included to clarify the probability area required. A common error was to include 84, and either use of the lower bound or no continuity correction with the normal formula. A number of solutions with correct method did not gain full credit due to premature approximation, either in calculating the variance/standard deviation or in the evaluation of the formula itself.

Question 5

(i) Stronger responses usually contained a sample space diagram of the possible outcomes before any probability calculations or the construction of the probability distribution table. The use of a sample space diagram aids the interpretation of the question information and enables the probabilities to be stated without any calculation. Several candidates interpreted the question as requiring only positive scores were permitted, and so either found the difference between the spinners or ignored the '-1' and continued with a probability total of less than 1. Candidates should be encouraged to read questions carefully, as solutions involving a red spinner numbered 1, 2, 3, 4 or the values on the spinners summed were often seen.

(ii) Most candidates who produced a probability distribution table in **5(i)** used the appropriate method here to calculate the variance. Better solutions clearly stated the unsimplified calculation for E(X) and then Var(X), with the final value stated as an accurate fraction. Where the probability distribution table was incorrect in **5(i)**, credit could be awarded for the method provided the unsimplified expressions were stated to justify the approach. A common error was not to use E(X)^2 in the variance calculation. Weaker solutions calculated E(X) and either stated that as the variance or squared this value. Premature approximation resulted in some answers being outside the required tolerance.

Question 6

(i) Many good solutions were seen for this Normal distribution question. Better solutions often included a simple sketch to help identify the required probability area. A common error was arithmetical inaccuracy when adding the value for the third decimal place of the z-value to state the probability. Most candidates correctly identified that the data was continuous and so did not use a continuity correction.

(ii) Many candidates did not attempt this question. Candidates who drew a sketch of the Normal distribution often identified that this question used the symmetry properties of the distribution with the answer from **6(i)** to calculate the required probability. However, most candidates calculated at least one of the probabilities within their solution. Final probabilities greater than 1 were common and these should have indicated to candidates that they have made an error.

(iii) The question was found challenging by many candidates. The most successful solutions were logically presented, with equations clearly identified and formed and algebraic processes accurately applied. The use of appropriate pure mathematic techniques is expected within this paper, and these should include sufficient working to justify the answer. Weaker candidates either found the z-values inaccurately, often using the tables to find a probability or using the original probability when forming the initial equations. The use of a sketch of a Normal distribution curve was seen in the stronger responses as this assisted in identifying if the z-value was negative.

Question 7

The use of simple diagrams throughout this question assisted candidates to interpret the conditions required.

(i) Many solutions did not apply the additional conditions identified in the instructions but calculated the total number of different ways that the 9 letters could be arranged. The use of a simple diagram assisted the best solutions to identify that there were only 6 different items to arrange, although some solutions did include additional terms to calculate the number of ways the block of Ts or Os could be arranged.

(ii) Many good solutions using both the subtraction or insertion approaches were seen. Better solutions using the insertion approach included a simple diagram to clarify the conditions, recognised that there was the effect of the 3 Os to eliminate as they were not identifiable and that when the Ts were inserted they could be interchanged and so needed to treated as a combination rather than a permutation. Better solutions using the subtraction approach clearly stated the total number of arrangements and arrangements with the Ts together being calculated, and these evaluated before the final subtraction. A common error was to not eliminate the effect of the repeated letters in either expression.

(iii) Candidates should be reminded to ensure they read the question carefully, as it was evident in this question that many candidates did not as few probabilities were attempted. Stronger solutions had a simple diagram to clarify the condition, and used similar approaches as the previous parts to calculate the values required within the probability. Most solutions simply calculated the number of arrangements with the Ts placed at the either end of the letters. Alternative approaches are possible, and some candidates successfully used conditional probability here.

(iv) Most candidates attempted this question, although few fully correct solutions were seen. Good solutions identified the possible scenarios that fulfilled the requirements before calculating the number of selections that were possible for each scenario. A common misconception was that if

OOT was chosen, then the remaining O and T would be included with the remaining letters for selection in the remaining places. Many candidates treated the Os and Ts as if they were individually identifiable and multiplied appropriately to include this within their calculation. The omission of OOOTT was not uncommon with a final total of 14 being stated.

MATHEMATICS

Paper 9709/72
Paper 72

Key messages

If candidates use the additional page or additional sheets for working, the question number must be clearly indicated.

It is important that candidates round answers to the number of significant figures as required in the question. Standard statistical notation must be used correctly for this component.

It is important that candidates understand how to recognise an underlying distribution and can also apply and justify a suitable approximating distribution when required to do so.

When carrying out a significance test the comparison between the test value and critical value, or equivalent, must be clearly shown in order to justify the conclusion. When making a conclusion to a significance test it should be in context and not definite.

General comments

There were some very good responses to this paper, however it was apparent that some candidates were not fully prepared for the demands of the examination. **Question 4**, in particular **parts (i)** and **(ii)**, were well attempted as was **Question 5**, whilst **Question 6** proved to be more challenging for candidates.

It is important that candidates can round correctly to 3 significant figures. There were occasions where only 2 significant figures were given with no indication of more accurate figures; this would result in not gaining accuracy marks.

There did not appear to be any time issues for candidates on this paper, and responses were generally well presented.

Comments on specific questions

Question 1

In this question, some candidates did not appear to understand the difference between the actual distribution of X and an approximating distribution. The given scenario was Binomial, $\mathrm{B}\left(500, \frac{1}{150}\right)$, which, because the relevant conditions were satisfied, could then be approximated to a Poisson distribution, namely $\mathrm{P}\left(\frac{10}{3}\right)$. The calculation in **part (iii)** should therefore have been done using a Poisson distribution. Many candidates used an incorrect Normal distribution in **part (iii)** or did not use an approximating distribution at all and found the probability using their Binomial distribution. Whilst some credit could be gained for this, it was not what was required by the question. It is important that candidates understand how to recognise an underlying distribution and if requested, as in this question, can then apply and justify an approximating distribution. When justifying an approximating distribution merely stating, for example, $np < 5$ is not sufficient; to fully justify the approximation it must be clear what np is equal to in the context given, so here it needs to be clear that $np = \frac{10}{3}$ and is less than 5. Some candidates approximating prematurely and using 3 or 3.3 for λ rather than $\frac{10}{3}$, therefore were unable to gain the relevant accuracy marks.

Question 2

In **part (i)(a)** many candidates omitted to give an assumption, or gave an incorrect one. The calculation to find *n* was generally well attempted, though some candidates did not realise that *n* should be an integer. Common errors included a sign error when setting up the initial equation and algebraic errors in rearranging this equation.

The null and alternative hypotheses should have been clearly stated when carrying out the test, and it was important here, in **part (i)(b)**, that the comparison between the test value and the critical value was clearly shown; this could be either as an inequality statement (e.g. $-2.182 < -1.751$) or on a clearly labelled diagram. Some diagrams drawn by candidates were not fully labelled and suggested, rather than stated, that -2.182 was in the rejection region without a full explanation of where the rejection region was. Some candidates recalculated the test statistic -2.182, often incorrectly, with many using their own incorrect figures from **part (i)**. It is important that the final conclusion to the test should be in context and not definite.

Few candidates gave a fully correct answer to **part (ii)**. It was important that the distribution that was identified as 'unknown' or 'not Normal' was clearly the Population. Responses such as 'It is not Normal' or 'the distribution is unknown' were not acceptable.

Question 3

Part (i) was reasonably well attempted. A minority of candidates did not calculate the unbiased estimate for the population variance and used the biased variance instead. There was confusion between the two formulae for the unbiased variance. Other errors included use of incorrect z values and not writing the answer as an interval.

Some candidates gave good answers for **part (ii)**, but many, although realising that the interval was wider, were unable to give a convincing reason. Some candidates thought the interval would be narrower.

Question 4

Parts (i) and **(ii)** of this question were well answered. In **part (i)** the most common method used was to integrate $f(x)$ and equate to 1. In **part (ii)** most candidates successfully integrated $xf(x)$ using correct limits and thus reached the required answer. **Part (iii)** was more challenging for candidates. The most common method was firstly to find the median by integrating $f(x)$ and equating to 0.5, where many candidates successful found that the median was $\sqrt{2}$ and then integrate between $E(X)$ and their median to find the required probability. Many candidates tried to integrate $f(x)$ but used incorrect limits. The more concise method, without finding the actual value of the median, was to find the probability of less than $E(X)$ then calculate 0.5 minus this probability; this method was rarely seen.

Question 5

Most candidates identified what was required in this question, although, on both parts of the question, some made errors when finding the variance. Other candidates, whilst finding the correct value for z were then unable to find the correct probability. Occasionally candidates made errors by confusing variance and standard deviation. In **part (ii)** some candidates did not achieve the required accuracy by rounding to 3 decimal places rather than the required 3 significant figures; if more accurate figures were not seen before rounding the candidate could not gain an accuracy mark.

Question 6

In **part (i)** the steps required, after stating the Hypotheses, were to calculate $P(X \geqslant 4)$ and $P(X \geqslant 5)$ and show that the first was >0.01 and the second <0.01. Many candidates found only the second of these probabilities, and some candidates did not answer to the 3 significant figure accuracy required. Other candidates, when attempting to find $P(X \geqslant 5)$ merely found $P(X = 5) + P(X = 6)$. Many candidates unnecessarily calculated $P(X \geqslant 6)$ in order to reach their conclusion.

In **part (ii)** it was important that the explanation of a Type I error was not a textbook definition but was put into the context of the question. Few candidates successfully found its probability.

Part (iii) was reasonably well attempted with some candidates identifying that they should calculate $P(X \geqslant 4)$ with $\lambda = 7.0$