MATHEMATICS

Paper 9709/12
Paper 12

Key messages

Now that papers are scanned in for marking it is very important that candidates use a black pen. If candidates write in pencil and then attempt to overwrite their work in pen their solutions are often very difficult to read. Similarly the work of those candidates who attempt to erase their pencil workings is very difficult to mark as the scanner picks up the original erased work as well as the new attempt. It is much better if rough working is not done separately but the whole attempt at a question is written in pen, in the space provided, and only diagrams are drawn in pencil.

Candidates should read the questions carefully at least twice and then extract the relevant information from them. Not fully comprehending the information often leads to candidates not gaining full credit.

General comments

The paper seemed to generally be very well received by candidates and many good and excellent scripts were seen. The paper contained a number of questions which were reasonably straightforward and gave all candidates the opportunity to show what they had learned and understood. It also contained some questions which provided more of a challenge, even for strong candidates. The vast majority of candidates appeared to have sufficient time to complete the paper.

Comments on specific questions

Question 1

This question proved to be a very accessible start to the paper, with many candidates demonstrating a good knowledge of the binomial expansion. Candidates were often able to write down the relevant term and evaluate it correctly. Weaker candidates sometimes did not include the minus sign or found the coefficient of $\frac{1}{x}$ instead of x.

Question 2

Most candidates demonstrated a very good understanding of the idea of a perpendicular bisector and full credit was very commonly gained in this question. Some weaker candidates obtained the wrong gradient for AB, did not find the gradient of the perpendicular, or didn't use the midpoint.

Question 3

In **(i)** most candidates realised the need to use the chain rule and did so correctly, although a significant number of candidates did not use the fact that the x-coordinate was decreasing and therefore obtained a positive, rather than a negative, solution. Weaker candidates sometimes differentiated again rather than using the $\frac{dy}{dx}$ given.

In **(ii)** the vast majority of candidates knew to integrate and then attempted to find the $+ c$. The mistakes that did occur were integrating incorrectly, forgetting about the $+ c$ or using the equation of a straight line instead of finding the equation of the curve.

Question 4

(i) in particular proved to be challenging for candidates. Many candidates knew that $\sin^2 x + \cos^2 x = 1$ was required and those who started with this statement rather than $a^2 + b^2$ were generally successful. In **(ii)** candidates who used the identity $\frac{\sin x}{\cos x} = \tan x$ to obtain an equation in a and b only, were usually successful, although some errors occurred in the subsequent re-arrangements.

Question 5

Many fully correct solutions to this question were seen, with candidates generally aware of the formula required for an arc length in radians and subsequently able to formulate the correct equation using the information provided in the question. Common errors were: using 180°, $\frac{\pi}{2}$ or 2π rather than π; forgetting to add the two radii for each sector; putting the 2 on the wrong side of the equation; and using the formula for area rather than perimeter.

Question 6

Those candidates who considered the given equations and thought about their likely shape were more successful in each part of this question than those who only substituted values. For example, for the curve, those who considered the cosine curve rather than just substituting 0 and 2π were more successful in **(i)**. Similarly re-arranging the line into the form $y = mx + c$ and then considering the y-intercept and gradient was the most successful approach. In **(ii)** many good curves and lines were drawn although some lines were not straight (a ruler should be used for straight lines) and some cosine curves did not appear to 'level off' at 0 and 2π. A significant number of candidates omitted **(iii)** although the majority did see the connection with the number of intersections of the graphs in **(ii)**.

Question 7

This question was very well answered by candidates, particularly finding the inverse of f in **(i)**, and solving the equation in **(ii).** In **(i)** weaker candidates did not obtain an expression for the inverse of g. Some candidates either didn't know how to obtain the value of x when $g^{-1}(x)$ was undefined or forgot to do so. In **(ii)** most candidates gained full credit although some weaker candidates evaluated $fg\left(\frac{7}{3}\right)$ or made mistakes in forming or solving the equation.

Question 8

(i) was very well done, with many candidates gaining full credit, although the other parts, particularly **(iii)**, proved to be more challenging. In **(i)** the vast majority of candidates successfully used the fact that when the given angle is 90°, the scalar product will be 0, although some candidates unnecessarily found the magnitudes of the vectors concerned. In **(ii)** most candidates used the magnitudes correctly although some did not consider the negative square root, or rounded incorrectly. Candidates found **(iii)** was more challenging and some candidates did not form the required vectors correctly, misinterpreted the given information, or did not form the unit vector once ***OC*** had been correctly found.

Question 9

This multi-stage question proved to be a real challenge for many candidates, although many fully correct solutions were also seen. Some candidates did not appear to have correctly interpreted the information given, and made incorrect steps such as equating the two curves or assuming that x was equal to 3 in the second curve as well as the first. Candidates who correctly interpreted the information given in the question often make significant progress. After differentiating the first curve there were a number of different correct methods that could be used, including: equating the tangent to the second curve and using the discriminant of the subsequent equation equated to 0 or differentiating the second curve and equating the two gradients. Some candidates' solutions were fully correct except that they did not find k or the coordinates of P. Candidates who drew a sketch of the situation were often successful and this approach is encouraged.

Question 10

It was important in this question, as always, to read the information given carefully. Many candidates who did this were able to formulate the correct equations in each part of the question and solve them. Common errors in the formulation were: in **(i)** making the sum of the first 10 terms equal to the sum of the first 5 or first 15 terms; in **(ii)** forming an equation where the 10^{th} term was 36 less, rather than 36 more, than the 4th term; and in **(iii)** multiplying the sum to infinity, rather than the sum of the first 4 terms, by 9. A number of candidates rounded prematurely in **(iii)** and did not obtain the correct final answer.

Question 11

Many completely correct solutions to this question were seen. In particular, **(i)** was very well done, with candidates successfully applying the standard rules for the differentiation and integration of this type of function. Candidates sometimes forgot to multiply and divide by 4 or did one but not the other. In **(ii)** most candidates realised that the stationary point was needed but many did not successfully solve their differential set to 0. A number of candidates did not use the fact that both terms in the equation were a function of $(4x + 1)$, so ended up with an equation that was difficult to solve. Those who did use this were often able to obtain and correctly solve a linear or quadratic equation. In **(iii)** candidates generally knew to use the integral from **(i)** but sometimes had the wrong limits or did not show all necessary working. When definite integrals are evaluated, it is important that both of the limits can clearly be seen to have been substituted into the integral. A significant number of candidates did not attempt this final part.

MATHEMATICS

Paper 9709/32
Paper 32

General comments

The candidate response to this paper was very varied. There were many candidates who offered strong responses to all ten questions, some candidates who showed a good level of understanding of the specification but lacked the arithmetic and algebra skills to complete the work accurately, and some candidates who demonstrated only a vague understanding of the topics examined.

The candidates scored particularly well in **Question 1** (binomial expansion), **Question 3** (trigonometric equation), **Question 7** (differential equation) and **Question 8** (partial fractions and integration). The questions that candidates found the most challenging were **Question 5** (complex numbers), **Question 6(i)** and **(ii)** (circular measure), **Question 9(ii)** (equation of a plane) and **Question 10** (trigonometric identity and calculus).

Key messages

- Show clear working.
- Write clearly and do not overwrite one solution with another. Overwriting solutions can result in an illegible script once scanned.
- Check your algebra and arithmetic carefully.
- Take care in your use of notation, and particularly in the use of brackets.
- If a question asks you to obtain a given answer then take particular care to show full working.
- If a question asks for an exact answer then decimal working is not appropriate.

Comments on specific questions

Question 1

The majority of candidates showed a good level of understanding of the binomial expansion. The most common error in expanding $(1+3x)^{\frac{1}{3}}$ was to use x in place of $3x$. Some candidates only expanded as far as the term in x^2, meaning that they had insufficient terms to complete their solution. Multiplication by $(3 - x)$ to find the coefficient of x^3 was often completed successfully. Several candidates stated the term in x^3 correctly but did not go on to state the coefficient of x^3.

A small number of candidates attempted the McLaurin expansion, which was a valid alternative method although involved rather more work and increased opportunities for arithmetic errors.

Question 2

Those candidates who recognised 9^x as $(3x)^2$ usually went on to form and solve a correct quadratic equation. Many candidates rejected the possibility of $3^x = -3$ and obtained the correct final answer. The usual ways to solve $3^x = 4$ were to start with $xIn3 = In4$ or to use $x = log_3 4$.

Candidates who did not recognise the equation as a quadratic in 3^x often started with an incorrect use of logarithms to obtain the incorrect equation $xIn9 = xIn3 + In12$. Another common error was to use $9^x = 3 \times 3^x$.

Question 3

The majority of candidates chose to rewrite the given equation as a quadratic in $tan\theta$, with many reaching the correct equation $tan^2\ \theta\ = \frac{1}{5}$. Some candidates did not then use the square root, and some only obtained one answer correctly because they did not consider both square roots. The alternatives of starting with $cot\ 2\ \theta = \frac{cos\ 2\theta}{sin\ 2\theta}$ and using the double angle formulae to form an equation in sinθ or in cosθ were quite common and usually successful.

Question 4

Most candidates applied the quotient rule or the product rule to find the derivative of $\frac{x}{1+ln\,x}$. The chosen rule was usually applied correctly. Many candidates simplified $\frac{1+ln\,x-\frac{x}{x}}{(1+ln\,x)^2}$ correctly, but there were a minority of candidates who 'cancelled' $(1\ +\ Inx)$ to obtain an incorrect answer such as $\frac{-1}{1+ln\,x}$. There was some confusion between the tangent and the normal, but several candidates reached the correct equation $\frac{ln\,x}{(1+ln\,x)^2} = \frac{1}{4}$. Only the stronger candidates went on to reach a correct quadratic in lnx. There were errors due to incorrect use of brackets and notation: $(lnx)^2$ was often written as ln^2x and this then became lnx^2. Some candidates who reached the correct quadratic in lnx then used the substitution $x\ =\ lnx$ to simplify their equation and stopped work when they had found a value for their x. Several candidates who obtained the correct solution for x then went into decimals so they did not obtain an exact value for y.

Question 5

(i) The majority of candidates were aware that the conjugate of the given root would also be a root of the equation. A few candidates changed both signs, and some did not include the i in their response.

(ii) The most popular approach was to start by substituting the given root into the equation to find the value of k. The question specifically ruled out the use of a calculator. Those candidates who simply wrote down $\left(-1+\sqrt{3}i\right)^3 = 8$, with no working shown, gained no credit. The common alternative approach was to use the two known roots to form the quadratic factor $x^2 + 2x\ +\ 4$ and then use the linear factor $kx\ +\ a$ to compare coefficients and solve for k. When successful, this approach was very efficient because it gave the third root at the same time. However, the common error was to combine the quadratic factor with a linear factor $x\ +\ a$, in which case no further progress was possible. Some candidates obtained the correct linear factor and stated that this was the root. Several candidates used their calculators to state the third root and gave no supporting working, so no marks were scored.

Question 6

(i) The simplest solution to this question was to use the isosceles triangle *OAB* to form two identical right angled triangles and then use $\frac{\frac{1}{2}AB}{r} = cos\,x$. Several candidates simply wrote down $\frac{AB}{2r} = cos\,x$ with no justification, and gained no credit. It was quite common to see longer solutions involving the sine rule or cosine rule. Many candidates offered no solution.

(ii) Candidates needed to equate half the area of the semicircle to the area of the sector. The given answer enabled several candidates to find their errors and correct their solutions. The most common errors occurred in the area of the sector, with many candidates using an angle of x rather than$2x$. There was a lot of evidence of misuse of factors of 2, with not all errors being traced back to their source. Some candidates who started with a correct equation lost the factor of x from the $16x$ in the denominator in the course of their working.

(iii) There were two common approaches used by candidates: either they considered an equation of the form $f(a) = 0$, or they worked with an equation of the form $f(a) = a$. Many candidates were unable to distinguish between the two forms and tackled the latter but were hoping to find a sign change. There were several calculation errors in combining the surd and the inverse trigonometric function. A few candidates worked in degrees rather than radians.

(iv) Many of the candidates who calculated values of the function correctly reached the correct solution having used the required level of accuracy. Candidates who used an initial value of 1.25 or 1.5 were more likely to reach the correct final answer than those who started from 1. This was largely due to the fact that candidates kept applying the iterative formula until two successive answers rounded to the same value correct to 3 decimal places. In cases where an initial value of 1 was used, the convergence was too slow for this to give the correct answer. When the convergence is slow, candidates should continue beyond the point where they think that they have found the root to ensure that the final figure is correct.

Question 7

(i) The majority of candidates started correctly, by separating the variables and attempting to integrate. The separation was usually completed correctly, and most candidates recognised the need to use integration by parts to integrate the function in x. Several candidates had $\int \frac{1}{e^y} dy$ and mistakenly believed this to be $ln(e^y)$. Those candidates who started with the form $\int e^{-y} dy$ were more likely to reach a correct answer. Many candidates went on to find the correct value for the constant of integration. The final step, going from an expression for $-e^{-y}$ to an expression for y was more challenging for some candidates in dealing with the negatives and using logarithms.

(ii) Candidates needed to be aware that $ln(1 - x)$ is only defined for values of x less than 1. Many answers were not expressed correctly, and it was commonly incorrectly stated that $ln(a)$ could not be negative.

Question 8

(i) Many candidates started correctly on the partial fractions. There were a few slips in the algebra and the arithmetic, but many fully correct solutions were seen. Candidates are advised to split the fraction as far as possible. Those candidates who opted for the partial form of $\frac{A}{2x+1} + \frac{Bx+C}{(2x+3)^2}$ could gain full marks in part **(i)** but they still had a lot of work to do before they could complete part **(ii)**.

(ii) Many candidates recognised the exact derivatives and integrated to obtain answers of the correct form. Those candidates who lost the 2 in the first log integral usually repeated the error in the second log integral. There were some sign and coefficient errors in integrating $(2x + 3)^{-2}$. It was rare for candidates who started with the two term form of the partial fractions to make any progress beyond integrating the first term. Many candidates with incorrect coefficients in their integration provided the correct answer, but the working often didn't contain sufficient detail to support the answer.

Question 9

(i) Many candidates were able to find the point of intersection of two lines. The majority of candidates who formed the equation of the line *AB* correctly went on to form and solve simultaneous equations and demonstrated clearly that the two lines did not intersect. There was one very common error: candidates who reached a correct equation $3\lambda = 2$ often concluded that $\lambda = \frac{3}{2}$.

(ii) Several candidates found half of the vector $\overrightarrow{AB}$ rather than the position vector of the midpoint of *AB*. It was generally understood that the direction vector of the line *AB* was perpendicular to *m*, and several candidates were able to use this, with their midpoint, to find an equation for *m*. Candidates who did not appreciate the significance of the line being perpendicular to the plane tried to use the vector product to find a normal vector, but did not start with directions in the plane so were not successful. Several candidates showed understanding of the correct method for finding the point of intersection of a line with a plane, and those who used all the information correctly usually reached the correct final answer.

Question 10

(i) Most candidates gave a correct expansion of at least one of $sin(3x + x)$ and $sin(3x - x)$. Some candidates then went on to use these correctly to derive the given result. Several candidates did not link their work to $sin4x$ and $sin2x$.

(ii) Although most candidates were aware of the need to find the value of $\int sin\,3\,x\,cos\,x\,dx$, many were unable to use the result of part **(i)** with this task. As well as completing the integration correctly, candidates also needed to determine the upper limit for their integral. It was common to see the incorrect value $\frac{\pi}{2}$ being used. Only the stronger candidates reached the final answer correctly. A number of candidates did not 'show all necessary working' and no marks were given for an answer that was not supported by correct working.

(iii) Many candidates understood that they needed to find $\frac{dy}{dx}$ in order to find the coordinates of *M*. Several did not follow the instruction to use the result of part **(i)** and started by differentiating the original form of the function. This never resulted in an equation in $cos2x$, but could have been followed through to find the x-coordinate of *M*. Several candidates who started with the correct form $2cos4x + cos2x$ rearranged this correctly to obtain an expression in $cos2x$, provided that they used a correct form of the double angle formula. Some candidates never obtained an equation in $cos2x$ because they used the double angle formula on both terms at the same time. If they followed this through, they could reach the final answer by forming a quartic in $cosx$. Several candidates who were solving a correct equation gave the final answer as the x-coordinate of the maximum point, rather than of *M*.

MATHEMATICS

Paper 9709/42
Paper 42

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper and cases where this was not adhered to were seen in **Question 1**, **Question 2**, **Question 3, Question 4** and **Question 5**. Candidates are advised to carry out all working to at least 4 significant figures if a final answer is required to 3sf.
- When answering questions involving an inclined plane, a force diagram can help candidates to ensure they have included all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable here in **Question 3** and **Question 4**.
- In questions such as **Question 7** in this paper, where velocity is given as a quadratic function of time, calculus must be used and it is not possible to apply the equations of constant acceleration.
- If an angle is given in terms of the sine, cosine or tangent of the angle then there is no requirement to evaluate the angle. When needed, all other trigonometric values can be obtained from the given information. There are examples of this in this paper in **Question 3**, **Question 4** and **Question 6**

General comments

The paper was generally well done by many candidates, although a wide range of marks was seen.

The presentation of the work was good in most cases and as the papers are now scanned, it is important to write answers clearly using black pen.

In **Question 3, Question 4** and **Question 6**, values of trigonometric functions were given and these can often lead to exact answers.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst differentiating well between even the stronger candidates. **Question 5** was found to be the easiest question whilst **Question 7(iii)** proved to be by far the most challenging.

One of the rubrics on this paper is to take g = 10 and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

This was a standard problem on force systems and the most straightforward approach was to resolve forces horizontally and vertically. This was the method used by the majority of candidates. Most candidates performed well on this question. Errors were seen due to the mixing up of sine and cosine particularly when evaluating components of the 17 N force. Another error often seen was accuracy being lost because of premature rounding. Candidates should again be reminded that in questions such as this they should keep all intermediate calculations to 4 or 5 decimal places in order to produce a final answer correct to 3sf. It was not possible to use Lami's theorem in this problem since there were 4 forces acting.

Question 2

This question involved motion that required the application of the constant acceleration equations. The most direct method, which was applied by the majority of candidates, was to apply the equation $s = ut + \frac{1}{2}at^2$ to the motion over the first 5 seconds using $t = 5$, and to apply the same equation for the first 8 seconds using $t = 8$. This gave a pair of simultaneous equations in the variables u and a. Solving these equations produced the required solution. An error that was seen was candidates attempting to apply the equation to the final 3 seconds of motion but wrongly using the same u as for the first 5 seconds. It was possible to apply the equation to the final 3 seconds but u had to be replaced by $u + 5a$. Another error that was seen was candidates misreading the given information, thinking that the 160 m referred to a distance in addition to the first 80 m, and applying the equation $s = ut + \frac{1}{2}at^2$ with $s = 240$ and $t = 13$. Overall this question was well done by the majority of candidates. Although it was possible to determine u and a exactly as fractions, some candidates used decimals and in some cases did not give answers correct to 3sf.

Question 3

This was a question in which the angle, θ, was given in terms of tan θ which meant that it was easy to find sin θ and cos θ exactly without calculating the angle. There were two different methods to approach this problem. In both cases it was necessary first to find the friction force. Resolving forces perpendicular to the plane gave the normal reaction, R, as $R = 13g \cos \theta$. Since motion was taking place $F = \mu R = 0.3 \times 13g \cos \theta$. One method of solution was then to resolve forces along the plane and since there was no acceleration, the force T N exactly balanced the friction force plus the component of the weight down the plane. This gave the value for T. Application of work done = $T \times 2.5$ gave the required answer. Another method was to use the work-energy principle which in this case stated that the work done by T was equal to the sum of the work done against friction and the gain in potential energy. Most candidates made a good attempt at this question. An error seen when using the relationship WD = $T \times 2.5$ was to wrongly also multiply this by cos θ. Some candidates made sign errors either when resolving forces or when combining work done against friction and potential energy and others mixed up sine and cosine of the angles.

Question 4

This was another question with the angle given in terms of sin θ and so it was not necessary to calculate the angle explicitly. There were two approaches that could be used here. Either Newton's second law could be applied to the motion of the car or alternatively the work-energy principles could be used. The majority of candidates chose to use Newton's second law. If this approach was adopted then there were three force terms to consider: the given driving force; the given resistive force; and the component of the weight down the plane. This combination of forces needed to be equated to $1250a$ and solved for a in both the up and down cases. An error seen when candidates applied this method was to omit one of the force terms or to use incorrect signs when combining them. Once the value of a had been found, application of the constant acceleration equations such as $v^2 = u^2 + 2as$ could be used to find the required speed. If the work-energy method was used then the equation consisted of 4 terms, namely the work done by the driving force, the work done against the resistance force, the change in potential energy and the change in kinetic energy. Both up and down motion involved the same terms but they needed to be combined with the correct signs, and errors in sign were the main cause of incorrect solutions. The kinetic energy term involved the required speed v and the equation could be solved to find this speed in each case. Another error that was frequently seen was the inclusion of a potential energy and a work done by the weight term, so including the potential energy twice. This question was well done by a large number of candidates.

Question 5

(i) This question included a given answer so candidates needed to show all of their working. Since the acceleration was required it was best to apply Newton's second law to both particles or to one particle and to the system. This produced two equations involving the tension, T, in the string and the acceleration, a, of each particle. Solving the equations gave the required results. Most candidates produced very good solutions to this question. An error that was seen on a few occasions was to think that the tension acting on particle A was different to the tension acting on particle B. Since the question asked to show the acceleration was $\frac{10}{3}$, it was important that candidates did not simply use their calculator to solve the equations as this is not a method for showing such a result and some detail of the method of solution had to be seen.

(ii) There were several methods available to find the maximum height of B. It was necessary to find the speed of particle B as particle A reached the ground and this could be achieved by using the equation $v^2 = u^2 + 2as$ with $u = 0$, $a = \frac{10}{3}$ and $s = 0.5$. Solving this gave the required speed, which then became the initial speed of the subsequent motion of B as it moved when the string became slack. Particle B then moved under the influence of gravity until coming to rest. Further use of the equation $v^2 = u^2 + 2as$ with u taken as the initial speed, $v = 0$ and $a = -g = -10$ and the extra height travelled after A reached the ground was found as $s = \frac{1}{6}$. The maximum height of B above the ground was found using $0.5 + 0.5 + \frac{1}{6}$. Many candidates made a good start to this part of the question but forgot to add the extra 0.5 m that particle B had travelled up to the point when A reached the ground.

Question 6

This question proved to be difficult for some candidates. Two situations were described in the question and it was necessary to write down the equations which described the motion in each case. The resistive force was given in terms of the velocity of the particle. In the first case, where the particle moved on a horizontal road, the speed was given as 18ms^{-1} and the driving force could be evaluated from the given information as $\frac{36000}{18} = 2000\text{ N}$. The driving force in this case was exactly balanced by the resistive force, leading to the equation of motion as 2000 = 18A + B. Many candidates did not substitute the value of $v = 18$ into their equation. In the second case the car moved up a hill with a given slope and in this case the driving force of $\frac{21000}{12} = 1750\text{ N}$ was balanced by a combination of the resistance and the component of the weight of the car down the slope. The value $v = 12$ had to be used and the resulting equation of motion took the form 1250 = 12A + B. These two simultaneous equations could then be solved for A and B. The candidates who substituted the relevant values of v into their equations found this to be a straightforward problem but a large number of candidates seemed to be confused by the fact that the resistance force was not constant and were reluctant to substitute a value for v. The condition $v > 2$ was given in the question, since this guaranteed that the resistance was positive. However, some candidates thought that it was necessary to apply this condition and several cases were seen where values of $v = 2$ and $v = 3$ were wrongly used.

Question 7

(i) Candidates were asked to sketch the velocity-time graph for the two particles. Although most candidates sketched the correct straight line graph for the motion of particle Q, a significant number of candidates chose to represent the quadratic expression for P as a set of connected straight lines rather than the correct quadratic curve passing through (0,0) and (3,0). In order to fully represent the nature of the motion it was vital to annotate the axes with values at the critical points and often this also was not seen.

(ii) There were several different ways of showing that P and Q met after 5 seconds. In all cases the displacements had to be found. For particle P this involved integrating the quadratic function. For particle Q, either integration could be used or the displacement at t = 5 could be evaluated by calculating the area under the straight line from $t = 0$ to $t = 5$. If integrals were used for both then one method was to evaluate the definite integral with limits of $t = 0$ and $t = 5$ and to show that both of these equated to 25. An alternative method was to equate the two integral expressions for the displacement of P and of Q. This led to a cubic equation which could be factorised as $t(t + 1)(t - 5) = 0$ and hence it could be shown that $t = 5$ is where the displacements were the same. Most candidates made a good attempt at this question. However some equated the two given velocities which was not what was required here but was an approach for **Question 7(iii)**.

(iii) This question proved to be the most difficult on the paper for almost all candidates. There were two different approaches which could be taken. Either the expressions for displacement found in 7**(ii)** could be subtracted to find the distance between the particles at any time or it could be realised that the maximum distance apart happened when the two velocities were equal. These two methods of approach are equivalent. If the distance between the particles was found, then this expression had to be differentiated and set to zero to find its maximum value. This was equivalent to setting the two velocities to be equal. This gave a quadratic equation in t which, when solved, gave the time at which the maximum distance occurred. It was found to be at $t = 3.19$ seconds (to 3sf). This value of t could then be used in the expressions for the two particles and the values subtracted to determine the maximum distance. Very few candidates scored full marks on this part. A common error was to think that the maximum occurred at $t = 3$. This was very close to the actual value but came from an incorrect method and hence did not score marks.

MATHEMATICS

Paper 9709/52
Paper 52

General comments

The work of most candidates was neat and well presented. A few candidates gave answers to 2 significant figures instead of 3 significant figures as requested. When giving answers to 3 significant figures this means working to at least 4 significant figures. Most candidates now use $g = 10$ as requested on the question paper.

The more challenging questions proved to be **Question 6(i)** and **(ii)**, and **Question 7(ii)** and **(iii)**. **Question 1**, **Question 3**, **Question 4(i)** and **Question 5(i)** were generally answered well.

Key messages

- Candidates should always refer to the formula booklet provided if in doubt of a particular formula.
- Answers should be given to 3 significant figures unless otherwise stated in the question.
- Write clearly and do not overwrite one solution with another.

Comments on specific questions

Question 1

(i) k could be found by comparing the given equation with the trajectory equation quoted in the formula booklet.

(ii) Again it was necessary to use the trajectory equation from the formula booklet.

(iii) Since the horizontal velocity remains constant $x = 28\cos 30 \times 3$.

Question 2

Many candidates were able to find the centre of mass from AB and/or AG but then did not use Pythagoras' theorem to find the required distance.

Question 3

This question was generally well answered.

(i) Candidates needed to find the radius and then use $F = mr\omega^2\omega^2\ \omega^2$.

(ii) This part of the question required candidates to resolve vertically.

Question 4

(i) This part of the question was generally well answered. It was necessary to use Newton's Second Law vertically.

(ii) Firstly, candidates needed to realise that the greatest downward speed occurred when the acceleration was zero. By putting $10 - 40x - 50x^2 = 0$, $x = 0.2$ could be found. Next it was necessary to integrate the equation found in part **(i)** with the constant of integration being zero. By substituting $x = 0.2$, the velocity could be found and hence the kinetic energy. Then, by using $EE = \frac{\lambda x^2}{(2l)}$, the elastic potential energy could be calculated.

Question 5

This question was generally well answered.

(i) Candidates needed to use $T = \frac{\lambda x}{L}$ twice and then solve the two equations to find a and λ.

(ii) By using $T = \frac{\lambda x}{L}$ and Newton's Second Law horizontally, the required speed could be calculated.

Question 6

(i) This part of the question proved to be challenging for many candidates. Candidates needed to find the horizontal and vertical velocities, v_H and v_V, when $t = 4$. At this point $(v_H)^2 + (v_V)^2 = 30^2$, resulting in an equation involving θ, which when solved gave the required answer.

(ii) Candidates needed to use $s = ut + \frac{1}{2}at^2$ vertically. When completed correctly this gave $s = -33.75$. Very few candidates explained that this meant below the level of projection.

Question 7

(i) A number of candidates found $r = 0.2$ from incorrect working. Some candidates used the incorrect formula for the centre of mass of the semi-circular arc.

(ii) This part of the question was challenging for many candidates. Candidates needed to take moments about A and so AC had to be found.

(iii) Many candidates found this challenging. Firstly, candidates needed to take moments about A. This resulted in the equation $0.8Y = (0.8 - 0.2\pi) \times \frac{AB}{2} + 0.2\pi \times AO$ where O is the centre of the semicircle and Y is the distance of the centre of mass from A. Using $AB = 0.8 - 0.2$ and $AO = 0.8 - 0.2 + 0.2$, Y could be found. Finally, $\tan\theta = \frac{0.1}{Y}$ could be used to find the required angle.

MATHEMATICS

Paper 9709/62
Paper 62

General comments

The majority of candidates presented their solutions in a logical manner. There was sometimes a lack of structure which made determining the final answer difficult for examiners.

Candidates often needed to show sufficient method to justify their conclusions. Not communicating intended processes produced uncertainty about the final answer. Greater accuracy often needed to be used within calculations, otherwise premature approximation affected the final answer.

Many good solutions were seen for **Question 3** and **Question 5**. The context in **Question 1** and **Question 4** was found challenging by many candidates. Management of time seems to have caused difficulties for some candidates to attempt all the questions.

Key messages

- Sufficient method must be shown to justify answers.
- Candidates should present their working in a clear manner.
- Only non-exact answers should be rounded to 3 significant figures.
- Referencing to context is important when determining the most appropriate approach for a solution.

Comments on specific questions

Question 1

A number of very clear solutions were seen for this question which used a standard context for throwing dice. Others needed to more clearly communicate the information required to determine independence, presenting work in a more methodical manner with annotations provided with the workings. The most efficient approach seen was to generate an outcome space initially and use this to identify the required probabilities. The best solutions indicated clearly on the outcome space values that fulfilled the conditions of Events S and T. Candidates needed to be able to interpret statements like ‘is less than 6’ as not including 6.

The main alternative approach was to list outcomes which fulfilled the events separately and then try to compare the lists for common terms. In both approaches, a significant number of counting errors were seen which led to incorrect probabilities. Weaker candidates often assumed independence and calculated $P(S \cap T)$ using $P(S) \times P(T)$. This is a ‘circular’ argument and gained little credit.

Question 2

Many good solutions to this normal approximation question were seen. The best solutions often had a simple sketch to identify the required probability area. Almost all candidates recognised that volume is a continuous variable, so no continuity correction was required.

A number of candidates were unable to answer the question completely as the calculation of the expected number of cartridges was omitted. Candidates needed to be aware that this value was an interpretation of their exact answer, rather than a rounding, so referencing approximation was inappropriate.

Question 3

Most solutions recognised that this was binomial distribution and used the appropriate formulae and approximations.

(i) The best solutions clearly stated the outcomes that would satisfy the condition, recognised that subtraction from 1 would be the most efficient method, and provided a clear unsimplified expression with efficient use of calculators to achieve an accurate 3 significant figures result. Weaker solutions often evaluated each term and, because of premature approximation, achieved an inaccurate final answer. A few candidates attempted to sum the probabilities of all of the scenarios, but these were often successful. Candidates needed to be able to interpret conditions such as 'at least 7' to indicate that only values higher than 7 were excluded. Some poor notation was noted, with the omission of the trailing bracket, or complete omission of brackets, but candidates frequently recovered when evaluating their answer.

(ii) Many good solutions were seen. These clearly stated the initial exponential equation, and then showed an appropriate method of solution. The use of logarithmic properties was acceptable and an efficient method. Candidates needed to ensure that their final answer fulfilled the stated condition.

Question 4

Many good attempts were seen to this question. The best solutions stated the z-values appropriately, used the standardisation formula accurately, and provided clear algebraic solutions of the simultaneous equations that were generated. A sketch of the normal distribution curve often assisted in identifying whether the z-value was positive or negative. A number of solutions rounded too early in the process, especially when μ was being eliminated and σ calculated first.

Question 5

The majority of candidates provided appropriate solutions to this question. Others needed to apply the context information more accurately to understand that a sweet would not be returned to the box. Re-reading the initial information after attempting each part of a question would have been helpful to ensure that the context had been appropriately applied.

(i) Almost all candidates attempted the tree diagram which is an extension of Upper Secondary mathematical knowledge and skills. The labelling was generally clear, although at times candidates were not consistent with the ordering of outcomes which caused difficulties later. It was not uncommon to have an unexpected additional branch between the first and second sweet removal branches where the changes to the box content were stated. Weaker responses often simply considered the original contents in a non-replacement context or continued the tree diagram to consider a third sweet being removed, which reduced possible credit. The use of a ruler for the branches was seen more frequently in solutions which were successful in later parts.

(ii) Almost all candidates produced a probability distribution table that was linked with the tree diagram that was created in part **(i)**. Weaker solutions ignored the context and considered possible outcomes of up to 6 toffees being removed. Candidates needed to be aware that the total of the probabilities for all the outcomes is 1, so that no table should sum to more than 1. Where the tree diagram was inaccurate, many gained partial credit using this property.

(iii) This part was omitted by some candidates who had created a probability distribution table. Candidates needed to be aware that 'mean' and 'expected value' were often interchangeable in this context. The best solutions stated the calculation that was being attempted before evaluating.

(iv) Most solutions identified that a conditional probability was being calculated. The best solutions often included a statement of the conditional probability formula for the context, used the tree diagram in part **(i)** to identify the required probabilities, and handled the fractions accurately. Many candidates were able to interpret their inaccurate tree diagram in part **(i)** appropriately and provided sufficient working to communicate their intentions to gain credit. Weaker solutions interpreted the denominator $P(T)$ as simply the probability for picking a toffee initially.

Question 6

(i) Many candidates found this question challenging. Understanding when using a box-and-whisker plot is appropriate is a requirement of the syllabus and candidates needed to be able to identify the key advantages and disadvantages of such a representation. The best responses often included several advantages including reference to the spread or shape of the data distribution as well as the more common ease of identifying the median or quartiles. The disadvantage was then normally related to the lack of other data present, so not being able to calculate the mean or mode were the more specific answers, although reference to the lack of individual data was also seen. Weaker answers often referred to the ease or difficulty of constructing the box-and-whisker plot. Uncertainty of the appropriate technical terms was noted, with the median often being referred to as the mean.

(ii) Whereas part **(i)** required a generic response, this part was placed in context and comments needed to relate to the specific data provided. Again, many candidates didn't show understanding of the appropriate technical terms, with median and mean frequently interchanged. Many candidates simply calculated the value of each measure of central tendency without making any conclusion. The best solutions identified that 768 was considerably larger than any other value thus making the mean inappropriate, and that there was no repeated value so effectively no mode, meaning that the median must be the most appropriate. Weaker answers often ignored the context and simply provided a reasoned approach with general descriptions of the median, mean and mode.

(iii) (a) Many solutions lacked the accuracy required at this level, with the use of inappropriate scales common. Candidates needed to ensure that any scale they used enabled them to communicate effectively their accurate values wherever possible. Most candidates found the median successfully, but the remaining quartiles were often incorrect. Good box-and-whisker plots were constructed using a ruler, ensuring that the diagram clearly communicated the intended value of the 5-points, with the whiskers being drawn at the middle of the box height and not entering the box itself. A linear scale using 1 cm for 20 minutes ensured accuracy and needed to be annotated with both the values linearly and labelled both 'time' and 'minutes'. Weaker solutions were often drawn freehand, omitted the labelling on the scale, and used a scale such as 2 cm for 37 minutes. Candidates needed to understand that at this level diagrams that represent statistical data need to be labelled with both units and the item that is being measured.

(b) Where the quartiles were identified in part **(iii) (a)**, most candidates were able to accurately calculate the interquartile range. Weaker responses often simply found the difference of the term values and restated the median as the answer.

Question 7

Most solutions recognised that this was a permutation and combination style question, and candidates were generally able to determine whether 'arrangements' or 'selections' were required by context. Candidates who presented their working in a clear manner were often more successful.

(a) The context for this part caused difficulties for some candidates. Successful solutions often had simple diagrams to visualise the problem, and this was a technique which provided support for some candidates. Most solutions considered loading the boats in the order stated in the question, although many solutions did not realise that once people were in a boat, they could no longer be placed in another boat, so that the pool of people being chosen from was reducing; ${}^6C_3 \times {}^6C_2 \times {}^6C_1$ was a common error. Some candidates calculated the ways of arranging the people in the boats, but then believed that the order that the boats were loaded needed to be considered, and amended their answer by multiplying by 6. Few candidates used permutations in their solutions.

(b) (i) Good solutions often included a simple 'diagram' to visualise the condition applied in the question. Most solutions realised the impact of the repeated value and divided as required. Weaker solutions often didn't multiply by 2 to allow for the order of 'odd' and 'even' to be interchanged.

(ii) Again, the best solutions included simple visual representations of the condition that was being applied. The more successful approach was to consider the arrangements of 3,7,7,7,8 and then determine how the 2s could be inserted between the numbers. A significant number of solutions didn't divide by 2 to remove the repeated value impact of the value 2.

Candidates who calculated the total number of ways the values could be arranged and then subtracted the number of ways where the 2s were together, were often less accurate. Many solutions didn't divide appropriately to remove the impact of the repeated values. A significant number of solutions didn't consider the 2s as a single item when calculating the number of ways that the values could be arranged, with the 2s together leading to some extremely complex and inaccurate approaches.

MATHEMATICS

Paper 9709/72
Paper 72

Key messages

It is important that candidates know how to round answers to the required number of significant figures (see, in particular, comments on **Questions 1** and **3** below)

If candidates use additional sheets for working, the question number must be clearly indicated.

Standard statistical notation needs to be used correctly.

Confusion between standard deviation and variance continues to prevent some candidates gaining marks (see **Question 2** below).

When carrying out a significance test, the comparison between the test value and critical value (or equivalent) must be clearly shown in order to justify the conclusion (see comments on **Questions 3** and **7** below).

General comments

This was a reasonably accessible paper for candidates. There were some very good scripts but also some scripts which demonstrated that candidates had found the paper very challenging. **Question 3** (particularly **part (i)**) was well attempted as was **Question 6**. Questions that proved to be more demanding were **Questions 2** and **7**.

As mentioned above, it is important that candidates can round correctly to 3.s.f. There were occasions where only 2 (or even 1) s.f. were given with no indication of more accurate figures; candidate who do this are unable to gain accuracy marks.

There did not appear to be any time issues for candidates on this paper, and presentation was generally good.

The comments below indicate common errors and misconceptions. However, many full and correct solutions were presented.

Comments on specific questions

Question 1

Question 1(i) was well answered, though many candidates gave their answer as 0.084 rather than 0.0842. If there was no greater accuracy seen here candidates did not gain the mark as 3.s.f. was required. Candidates were either confusing significant figures with decimal places, or thought that the zero after the decimal point was significant.

Parts (ii) and **(iii)** were reasonably well attempted, though manipulation of factorial notation and indices caused confusion for weaker candidates.

Question 2

Candidates found this question very challenging, particularly **part (ii)**.

In **part (i),** some candidates did not describe the distribution (Normal) fully and did not give the parameters. Others confused standard deviation and variance, correctly working out one, but stating it was the other. Standard notation for the Normal distribution was not always used correctly here. There were also candidates who did not know what the question required.

Very few candidates gave a correct answer in **part (ii)**. Statements such as 'it should be Normal' were quite common but did not gain credit. Candidates had to have made it clear that the population (i.e. X here) was Normal in order for the distribution to be exact. Incorrect answers involving the central limit theorem or values for n were often seen.

Question 3

Part (i) was particularly well attempted, with the majority of candidates able to correctly calculate unbiased estimates of the mean and variance. Confusion between the two formulae for the unbiased variance was not as prevalent as has been the case in the past, with the majority of candidates using the formula given on the formula sheet. Very few candidates gave the biased estimate of the variance. Once again, there was confusion by some candidates in giving a final answer correct to 3 significant figures.

There were many full and complete solutions for **part (ii)**, but in some cases the hypotheses were omitted or incorrect and errors were made when standardising (for example not using $\sqrt{50}$). In order to justify the conclusion, a comparison between the test value and the critical value had to be clearly shown either as an inequality statement or on a fully labelled diagram; some candidates did not show this comparison. It is not sufficient to merely state 'compare'. The conclusion to the test should be written in context and should not be definite.

Question 4

This question was reasonably well attempted. Many candidates scored highly, though few candidates gain full credit. Errors included using an incorrect variance, incorrectly using a continuity correction or calculating an incorrect area (> 0.5 rather than < 0.5). Very few candidates realised that their calculated probability needed to then be doubled for the final answer.

Question 5

Part (i) was usually well attempted, though 'p' was sometimes omitted, or λ or μ incorrectly used. Weaker candidates did not always realise what was required. In **part (ii)** candidates often did not choose the required Binomial distribution $B(40,0.1)$. Whilst the confidence interval in **(iii)** was generally well attempted, there were candidates who either incorrectly used 0.1 and 0.9, or did not work with proportions. **Part (iv)** required candidates to check if 0.1 was in their CI and make a conclusion from this. Comments such as 'it' lies in the CI are too vague and did not gain credit, and some candidates did not give both the required parts of the answer.

Question 6

Questions on probability density functions are usually well attempted and this question was no exception. In **part (i)** most candidates realised that the requirement was to integrate $f(x)$ with limits 1 and b and equate to 1. The resulting equation in 'a' and 'b' was usually correctly rearranged to the form given. **Part (ii)** was also well attempted, though weaker candidates confused the values of 0.5 and the median. Instead of integrating from 1 to 1.5 and equating to 0.5, the integral was incorrectly evaluated from 1 to 0.5 and equated to the median 1.5. In **part (iii)** it was required to integrate $xf(x)$ with limits 1 and b. This was usually well attempted, though weaker candidates were unable to correctly carry out the integration of $xf(x)$ (i.e. $\frac{1}{x}$).

Question 7

This question was found to be very challenging by candidates. In **part (i)** many candidates did not fully understand that 320 was the number of passengers who bought tickets, or, alternatively, the number of seats on the plane. It was not simply the number of passengers or, as many candidates thought, the number of passengers who bought tickets but did not turn up. A Poisson distribution with $\lambda = 3.2$ was required in **part (ii)**. Errors included an incorrect value for λ, an extra term, or a missing term, in the expression for $P(3,4,5)$ or even an incorrect expression involving $P(< 6)$ and $Pr(> 2)$. The justification for using the Poisson approximation in **(iii)** was often not fully correct or was not in context. It was not sufficient to just say $np < 5$, np needed to be stated as 1.6 to be 'in context'. **Part (iv)** was poorly attempted, with candidates using a range of methods: from the correct Poisson distribution to incorrect Poisson distributions or more usually trying to incorrectly model the situation with an invalid Normal distribution. Some candidates only included one term in their (correct form of) calculation and lost marks accordingly. The final conclusion for the test should be fully justified, written in context and not definite.