



Cambridge O Level

ADDITIONAL MATHEMATICS

4037/01

Paper 1

For examination from 2020

MARK SCHEME

Maximum Mark: 80

Specimen

This document has **12** pages. Blank pages are indicated.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

<p>GENERIC MARKING PRINCIPLE 1:</p> <p>Marks must be awarded in line with:</p> <ul style="list-style-type: none"> ● the specific content of the mark scheme or the generic level descriptors for the question ● the specific skills defined in the mark scheme or in the generic level descriptors for the question ● the standard of response required by a candidate as exemplified by the standardisation scripts.
<p>GENERIC MARKING PRINCIPLE 2:</p> <p>Marks awarded are always whole marks (not half marks, or other fractions).</p>
<p>GENERIC MARKING PRINCIPLE 3:</p> <p>Marks must be awarded positively:</p> <ul style="list-style-type: none"> ● marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate ● marks are awarded when candidates clearly demonstrate what they know and can do ● marks are not deducted for errors ● marks are not deducted for omissions ● answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
<p>GENERIC MARKING PRINCIPLE 4:</p> <p>Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.</p>
<p>GENERIC MARKING PRINCIPLE 5:</p> <p>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).</p>

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, given for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the **M** marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several **B** marks allocated. The notation ‘dep’ is used to indicate that a particular **M** or **B** mark is dependent on an earlier mark in the scheme.

Abbreviations

AG	answer given
awrt	answer which rounds to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	special case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(a)	$2(2)^3 - 3(2)^2 + 2q + 56 = 0$ with one correct interim step leading to $q = -30$	1	For convincingly showing $2(2)^3 - 3(2)^2 - 30(2) + 56 = 0$ or correct synthetic division at least as far as $\begin{array}{r rrrr} 2 & -3 & q & 56 & \\ & 4 & 2 & 2q & +4 \\ \hline & 2 & 1 & q+2 & 0 \end{array}$ then $q = -30$ or correct long division to, e.g. verify -30 , at least as far as: $\begin{array}{r} 2x^2 + x - 28 \\ x - 2 \overline{) 2x^3 - 3x^2 - 30x + 56} \\ \underline{2x^3 - 4x^2} \\ x^2 - 30x \\ \underline{x^2 - 2x} \\ - 28x + 56 \\ \underline{-28x + 56} \\ 0 \end{array}$
1(b)	$2x^2 + x - 28$ oe	B2	For any two terms correct
	$(x - 2)(2x - 7)(x + 4)$ oe	M1	For factorising the correct polynomial
	$x = 2, x = -4, x = 3.5$ oe	A1	Answer only scores 0.

Question	Answer	Marks	Partial Marks
2(a)	$\left[\frac{dy}{dx} = \right] \frac{3}{2}\sqrt{x}$ oe	B2	Allow unsimplified, e.g. $\left[\frac{dy}{dx} = \right] x\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + x^{\frac{1}{2}}$ from product rule B1 for $y = x^{\frac{3}{2}}$ or for one correct term in the sum obtained using the product rule
2(b)	$[y = 8]$ $x = 4$	B1	
	$\frac{0.015}{\delta x} \approx \left(\text{their } \frac{dy}{dx}\right)_{x=4}$ oe	M1	Condone $\frac{0.015}{\delta x} = \left(\text{their } \frac{dy}{dx}\right)_{x=4}$
	0.005 oe nfw	A1	

Question	Answer	Marks	Partial Marks
3(a)	$12\left(x - \frac{1}{4}\right)^2 + \frac{17}{4}$ isw	3	B1 for each of p, q, r correct; Allow correct equivalent values If 0 scored, SC2 for $12\left(x - \frac{1}{4}\right) + \frac{17}{4}$ or SC1 for correct 3 values but incorrect format
3(b)	$\frac{4}{17}$ is greatest value when $x = \frac{1}{4}$	2	Strict FT <i>their</i> $\frac{17}{4}$ and <i>their</i> $\frac{1}{4}$ B1 for $\frac{4}{17}$ and B1 for $x = \frac{1}{4}$ Each value must be correctly attributed; Condone $\left(\frac{1}{4}, \frac{4}{17}\right)$ for B2 Condone $y = \frac{4}{17}$ for B1

Question	Answer	Marks	Partial Marks
4	$AX = \sqrt{45}$ soi	B1	May be implied by $3\sqrt{5}$
	$AX = 3\sqrt{5}$	B1	May be seen later
	$\frac{1}{2}(4 + \sqrt{5} + 2 + x) \times \text{their } \sqrt{45}$	M1	May be implied by, e.g. summation of rectangle and two triangles
	$15(\sqrt{5} + 2) =$ $\frac{1}{2}(4 + \sqrt{5} + 2 + x) \times \text{their } \sqrt{45}$ or better	M1	Must be correct apart from <i>their</i> $\sqrt{45}$
	Correctly divide <i>their</i> equation by <i>their</i> $\sqrt{5}$ or <i>their</i> $\sqrt{45}$ and rationalise denominator	M1	or correctly multiply both sides of <i>their</i> equation by <i>their</i> $\sqrt{5}$ or <i>their</i> $\sqrt{45}$ and obtain a rational coefficient of x soi
	Completion to $4 + 3\sqrt{5}$ nfw	A1	

Question	Answer	Marks	Partial Marks
5(a)	Correct shape for both graphs	B2	B1 for either Must touch the x -axis in the correct quadrant
	Correct y -intercept for both graphs and Correct x -intercept for both graphs	B2	B1 for either $y = 2$, $y = 5$ or $x = 2$, $x = -2.5$ or $y = 2$, $x = -2.5$
5(b)	$2x + 5 = \pm(2 - x)$ oe or $(2x + 5)^2 = (2 - x)^2$	M1	For attempt to obtain 2 solutions; must be a complete method
	$x = -7, x = -1$	A1	
	$-7 \leq x \leq -1$	A1	FT <i>their</i> values of x

Question	Answer	Marks	Partial Marks
6	$\frac{1}{2}(2x)(x^2 + 5)^{-\frac{1}{2}}$ for quotient seen or $-\frac{1}{2}(2x)(x^2 + 5)^{-\frac{3}{2}}$ for product seen	B1	
	Correct application of the quotient rule or product rule attempted	M1	
	$\frac{dy}{dx} = \frac{2(x^2 + 5)^{\frac{1}{2}} - \frac{1}{2}(2x)(x^2 + 5)^{-\frac{1}{2}}(2x - 1)}{x^2 + 5}$ or $\frac{dy}{dx} = 2(x^2 + 5)^{-\frac{1}{2}} - \frac{1}{2}(2x)(x^2 + 5)^{-\frac{3}{2}}(2x - 1)$	A1	All correct; may be unsimplified
	$x = 2, y = 1$ $\frac{dy}{dx} = \frac{4}{9}$	B2	B1 for each
	Gradient of normal = $-\frac{9}{4}$	M1	Valid attempt to obtain gradient of normal
	$y - 1 = \textit{their} \left(-\frac{9}{4}\right)(x - 2)$ oe	M1	
	$9x + 4y = 22$	A1	

Question	Answer	Marks	Partial Marks
7(a)	$20 = \pi x^2 + xy$	B1	
	$y = \frac{20 - \pi x^2}{x}$	B1	
	$P = 2\pi x + 2x + 2y$ $= 2\pi x + 2x + 2\left(\frac{20}{x} - \pi x\right)$	M1	Attempt to use perimeter and obtain in terms of x only
	$= 2x + \frac{40}{x}$	A1	For all steps seen, nfwfw AG
	Alternative $20 = \pi x^2 + xy$ and $P = 2\pi x + 2x + 2y$	B1	
7(a)	$P = \frac{2}{x}(\pi x^2 + xy) + 2x$	M1	For attempt to use perimeter and write in $\frac{\pi x^2 + xy}{x}$
	$= \frac{2}{x}(20) + 2x$	B1	For replacing $\pi x^2 + xy$ with 20
	$= 2x + \frac{40}{x}$	A1	For all steps seen, nfwfw AG
	$\frac{dP}{dx} = 2 - \frac{40}{x^2}$ When $\frac{dP}{dx} = 0$ $x = 2\sqrt{5}$ or 4.47 or $\sqrt{20}$ Leading to $P = 8\sqrt{5}$ or 17.9	5	M1 for attempt to differentiate DMI for equating to zero and attempt to solve at least as far as $x^2 = \dots$ A1 for x A1 for P A1 for this statement or use of gradient inspection either side of correct x
	$\frac{d^2P}{dx^2} = \frac{80}{x^3}$ Always positive so a minimum		
7(b)	oe		

Question	Answer	Marks	Partial Marks
8(a)(i)	ke^{5x-1}	M1	
	$k = \frac{1}{5}$ oe	A1	
	$\frac{1}{5}(e^{5(1)-1} - e^{5(0.2)-1})$ or better	M1	
	$\frac{1}{5}(e^4 - 1)$ or $\frac{1}{5}e^4 - \frac{1}{5}$	A1	Answer only scores 0/4.
8(a)(ii)	Expands correctly	B1	May be unsimplified
	$\left[\frac{1}{x} dx = \ln x \right]$ soi	B1	
	$\left[\frac{x^3}{3} + 2 \text{ their } \ln x - \frac{x^{-3}}{3} \right]$	M1	Integration of their 3-term or, if unsimplified, 4-term expression
	$\left[\frac{2^3}{3} + 2 \text{ their } \ln 2 - \frac{2^{-3}}{3} \right] - \left[\frac{1^3}{3} + 2 \text{ their } \ln 1 - \frac{1^{-3}}{3} \right]$	M1	Condone omission of lower limit
	$2 \ln 2 + \frac{21}{8}$ oe	A1	Any equivalent answer that is simplified to two terms Answer only scores 0/5.
8(b)	$k \cos\left(\frac{x}{6}\right) (+c), k < 0$	M1	
	$-6 \cos\left(\frac{x}{6}\right) + c$	A1	

Question	Answer	Marks	Partial Marks
9	$\frac{n(n-1)(n-2)(n-3)(2^4)}{4 \times 3 \times 2 \times 1}$ $= 10 \frac{n(n-1)(2^2)}{2 \times 1} \text{ or better}$	M3	Condone omitting the factor of n and/or $n-1$; must have dealt with factorials M2 if one slip/omission or M1 if two slips/omissions or B1 for $\frac{n(n-1)}{2}(2)^2[x^2]$ seen and B1 for $\frac{n(n-1)(n-2)(n-3)}{24}(2)^4[x^4]$ seen
	$n^2 - 5n - 24 [=0]$ oe	A1	Equivalent must be 3 terms, e.g. $n^2 - 5n = 24$
	$(n+3)(n-8) [=0]$	M1	or any valid method of solution for <i>their</i> 3-term quadratic
	$n = 8$ only	A1	A0 if -3 also given as a final solution, i.e. not discarded
Question	Answer	Marks	Partial Marks
10(a)	$-200 > \frac{n}{2}(-10 + (n-1)(-3))$ leading to $3n^2 - 13n - 400 (> 0)$ $n = 13.9\dots$ so 14th term needed	4	M1 for attempt to use sum to n terms, allow use of $=$ or \leq or $<$ A1 for correct quadratic expression DM1 for attempt to solve A1 for correct conclusion
10(b)(i)	$ar^2 = \frac{81}{64}$ $ar^4 = \frac{729}{1024}$ $r^2 = \frac{9}{16}$ $r = \frac{3}{4}$ $a = \frac{9}{4}$	5	B1 for 3rd term B1 for 5th term M1 for attempt to solve their equations to obtain either r or a A1 for r A1 for a
10(b)(ii)	$S_\infty = 9$	B1	FT on <i>their</i> a and r , provided $ r < 1$

Question	Answer	Marks	Partial Marks (Separate areas subtracted)
11	Method 1		
	$[x_B = x_C = 5 \text{ soi}]$	B1	
	$\left[\int (x^2 - 4x + 10) dx = \right]$ $\frac{x^3}{3} - \frac{4x^2}{2} + 10x$	M2	or M1 for at least one term correct
	Correct or correct FT substitution of limits 0 and <i>their</i> 5 into <i>their</i> $\left[\frac{x^3}{3} - \frac{4x^2}{2} + 10x \right]$	M1	dep on at least M1 being earned; Condone + c as long as <i>their</i> c is not numerical
	$\frac{1}{2} (10 + 15) \times 5 \text{ oe}$ or $\int_0^5 (x + 10) dx = \left[\frac{x^2}{2} + 10x \right]_0^5$ $= \frac{(5)^2}{2} + 10(5) \text{ oe}$	B2	or M1 for $\frac{1}{2}(\text{their } 10 + \text{their } 15) \times \text{their } 5 \text{ oe}$ or B1 for $\int (x + 10) dx = \frac{x^2}{2} + 10x$
	<i>their</i> $\left(\frac{125}{2} - \frac{125}{3} \right)$	M1	
$\frac{125}{6}$ or $20\frac{5}{6}$	A1	Answer only scores 0/8.	

Question	Answer	Marks	Partial Marks (Subtracting and using integration once)
11	Method 2		
	$[x_B = x_C =]5$ soi	B1	
	$\int (-x^2 + 5x) dx$	B1	Condone omission of dx
	$\left[-\frac{x^3}{3} + \frac{5x^2}{2} \right]$ oe or $\left[\frac{x^3}{3} - \frac{5x^2}{2} \right]$ oe	M3	or M2 for $\int (px^2 + qx) dx = \frac{px^3}{3} + \frac{qx^2}{2}$ oe either with $p = \pm 1$ or $q = \pm 5$ or M1 for $\int (px^2 + qx) dx = \frac{px^3}{3} + \frac{qx^2}{2}$ with non-zero constants p and q , with $p \neq \pm 1$ and $q \neq \pm 5$
	Correct or correct FT substitution of limits 0 and <i>their</i> 5 into <i>their</i> $\left[-\frac{x^3}{3} + \frac{5x^2}{2} \right]$	M2	Condone omission of lower limit
$\frac{125}{6}$ or $20\frac{5}{6}$	A1	Answer only scores 0/8.	