ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

It is important that candidates are familiar with the rubric on the front of the examination paper as it will remind them of important facts which tend to be forgotten during the examination itself. It is important that the degree of accuracy is noted as many candidates lose accuracy marks due to inaccurate working. It is also important that candidates ensure that they have fulfilled the demands of each question and have also set their work out clearly, showing sufficient steps in their working.

General comments

The majority of candidates appeared to have sufficient time to work through the paper and also sufficient space in which to answer their questions. It was pleasing to note that when extra space was needed for an answer, work was done on additional sheets which were invariably annotated with the appropriate question number.

It should also be noted that many candidates did not obtain some of the accuracy marks available as they did not appreciate the implication of the word ‘exact’ in the mathematical context. It should also be noted that when candidates are required not to use a calculator, this instruction extends to the whole question and not just the first two or three lines of a solution.

Comments on specific questions

Question 1

(a) The majority of candidates scored full marks by correctly shading both Venn diagrams. Those that made an error tended to make it on the shading of the Venn diagram for $A' \cap B' \cap C'$.

(b) This part proved to be more problematic for candidates, with many not providing all the elements from 0° to 360° for set *P*. It was common to see just the two elements in the range 0° to 180° rather than the four elements in the range 0° to 360°, although most candidates dealt with the double angle correctly. Some candidates gave just one element for each of sets *P* and *Q*.

It was acceptable for the elements of sets *P* and *Q* to be listed and most candidates interpreted the intersection of their sets correctly. However, correct set notation was required for the final answer as a set was asked for. Many candidates listed the elements of the intersection, although the elements $30°$ and $150°$ were often obtained fortuitously from an incorrect set *Q*. These candidates were usually only able to gain one mark.

Question 2

It was essential that candidates appreciated that the use of a calculator was not allowed in any part of this question.

The solutions offered fell mainly into 3 types:

The first type involved candidates equating the two equations and cancelling out $2x+3$ from each side of their equation. The remaining quadratic equation was then solved by factorisation, with the cancelled factor of $2x+3$ being ignored, thus restricting candidates to 3 marks at best.

The second type involved candidates equating the two equations as before and forming a single cubic equation, equated to zero. However many candidates then immediately produced the roots of this equation, showing no intermediate working, which was not the required (non-calculator) method. Such candidates were only able to score a mark for the initial equating of the two equations.

The third type involved a correct method with candidates producing a cubic equation equated to zero and then showing a method for testing for a root or factor. The cubic equation could then be fully factorised first producing a linear factor and a quadratic factor which then led to the production of three *x*-coordinates which usually (but not always) were used to find corresponding *y*-coordinates. Occasionally the coordinates were only found for two points, occurring when the candidates attempted to make use of the quadratic factor $4x^2-9$, producing only $x=\frac{3}{2}$. Sometimes the root from the first factor was ignored.

Very few candidates produced the solution that had been envisaged with the two equations being equated and then factorisation using the common factor of $2x+3$ taking place.

Question 3

(i) Nearly all candidates obtained the correct answer of 1000.

(ii) Most candidates were able to differentiate the exponential expression correctly, with just a very small number trying to subtract one from the powers of the exponential terms. These exponential terms were then equated to 1200 and correctly divided throughout by 400. Even though the final answer was given, a small number of candidates were not able to complete the solution by multiplying their correct result of $e^{2t}-4e^{-2t}-3=0$ throughout by e^{2t}.

(iii) Many completely correct solutions were seen, but some candidates did not check that they had completely answered the question. The most common error was for candidates to use the given substitution and solve the resulting quadratic equation to give two values for *u*, without completing to find *t* as required. The final answer was acceptable in either logarithmic or decimal form. However, a few candidates gave their final answer for *t* as 0.69 rather than the expected decimal answer of 0.693. Most candidates realised correctly that the equation $e^{2t}=-1$ does not provide a solution. Those that gave an erroneous solution for this equation did not gain the final accuracy mark as it depended on having a correct solution and no other.

Question 4

(a) The majority of candidates were able to find the correct value for each of the unknown indices. Very few candidates did not get any correct. Some candidates calculated the powers of *p*, *q* and *r* but did not always match them up correctly with the *a*, *b* and *c* required by the question. Such candidates were awarded the marks provided a correct simplification had been seen.

(b) Many candidates were able to find a successful method for dealing with the given simultaneous equations, often by substituting a pair of letters for $\sqrt{x}$ and $\sqrt{y}$, and then solving in the usual way, Those candidates that chose to use *x* and *y* as the substitute pair often confused themselves later. One of the most common incorrect methods was to square each term in each equation producing the incorrect $9x - y = 16$ and $16x - 9y = 196$. Another common error was to forget to multiply every term in one of the equations in preparation for elimination of one variable. Quite often candidates found having the correct square roots as the unknowns had problems handling them, so, for example, $\sqrt{x} = 2$ would become $x = \sqrt{2}$. Similar misapplications during the process of eliminating a variable produced much unnecessary and incorrect work from some candidates.

Question 5

The majority of candidates used radians, as intended, throughout this question.

(i) Most candidates made use of the arc length formula to obtain a correct angle in radians. Very few incorrect results were seen.

(ii) The correct two areas needed to produce a correct solution were identified by most candidates with many going on to find these areas correctly. The better prepared candidates were able to obtain the areas needed by using either $\tan 0.8$ or $\cos 0.8$ as intended. Some candidates found the triangle area without presenting and/or working to the correct degree of accuracy for the length of either *OB* or *AB*. Candidates who rounded these lengths to 3 significant figures before finding the area of the appropriate triangle at this stage obtained a final area that was inaccurate, thus being unable to obtain the final accuracy mark.

Other methods, such as use of the sine rule to find the lengths needed were also acceptable. Unfortunately some candidates mistakenly assumed, from the diagram, that the triangle was isosceles. If this was the case, the only mark available was for the area of the appropriate sector. Candidates should be guided to some extent by the mark allocation for a question as to how much work is involved in the solution of that question.

Question 6

Most candidates were able to identify that part **(a)** of this question involved the use of permutations and part **(b)** of this question involved combinations.

(a) (i) The majority of candidates were able to find the correct solution.

(ii) Most candidates realised the number of arrangements of the mathematics books within themselves was 4!, but then did not always treat them subsequently as one unit. This meant that multiplying by 120 was not always done. Rearranging the remaining 4 books and then doubling was a common error so 4! x 4! x 2 was a frequent incorrect response. A few candidates attempted to use $^4C_4$ for the arrangement of the mathematics books without success.

(iii) There were similar errors to those in part **(ii)**. Many candidates realised that 4! x 3! was needed for the number of ways the mathematics books can be arranged amongst themselves and the number of ways the geography books can be arranged amongst themselves, Unfortunately, this was often multiplied by 3 or 4 suggesting that the number of arrangements was being counted instead of using 3! or the equivalent.

(b) (i) The majority of candidates were able to find the correct solution.

(ii) Many candidates were able to produce a correct solution by finding all the combinations that included 1, 2, 3 or 4 women in the team rather than the more efficient route of finding how many teams were available with only men and subtracting that from their solution for the previous part – this process would have been less prone to some of the arithmetical slips or omissions that were liable to occur in the longer method.

Question 7

The greatest problem that candidates had dealing with this question was being able to interpret the information in order to draw a suitable triangle to start to work with.

(i) Many candidates were unable to draw a correct vector triangle. The basic understanding of bearings was generally sound, with *AB* usually drawn correctly. However, many diagrams were simply right-angled triangles with *AB* as the hypotenuse. It was also a common error to label the path *AB* as being at 650 kmh^{-1}. Those candidates who drew correct diagrams usually realised that triangle side lengths are in proportion to speeds and that the west to east wind assists the plane. However, wind direction was often shown as east to west so that the plane's course was the longest side of an obtuse-angled triangle. This gave a correct angle at *B* of 6.08° but incorrect bearing and speed for the plane. Some candidates drew acute-angled triangles with no regard to relative speeds and lengths and, although they were able to get an angle of 6.08°, were then unable to use their triangles to convert this angle to the correct bearing. Candidates were adept at applying both sine rule and cosine rule to their diagrams to find angles and speeds.

(ii) A variety of methods were used to find the resultant speed of the plane going *A* to *B*. As in part **(i)**, the majority of candidates used the cosine rule (or Pythagoras when they had right-angled triangles) for the resultant speed. Others misused vector arithmetic and produced a speed of 770 kmh^{-1} or 530 kmh^{-1}. Candidates very rarely confused speeds and distances in their diagrams, and were clear in their use of distance and speed to find the journey time. A few candidates calculated the actual distance travelled by the plane and divided by 650 for the time taken. From correct diagrams there were some errors in calculating the time taken due to inappropriate rounding of angles and/or speeds in the calculations, or simply due to careless rounding of the final answer.

There were, however, some exemplary solutions from capable candidates.

Question 8

(i) Many completely correct expressions were seen for e^y in terms of *x*. However, some candidates made errors in the final stage of their working when attempting to use logarithms. Many completely correct expressions for *y* were also seen. Most candidates realised that an equation of the form $e^y = \frac{m}{x} + c$ connected the variables *x* and *y*. Some candidates misused the given coordinates in the equation $e^y = \frac{m}{x} + c$ and thus obtained incorrect values for *m* and for *c*. Other candidates found the gradient of the straight line, *m*, correctly but then used a non-linear form to attempt to find *c*.

(ii) Although many candidates had a correct expression for *y* in part **(i)**, few were able to find the correct values of *x* for which *y* is defined. Of those that made a reasonable attempt, some made an error with the inequality sign or used $32 - \frac{6}{x} > 1$ rather than $32 - \frac{6}{x} > 0$.

(iii) Correct solutions were often seen from those candidates who had a correct expression for *y* in part **(i)**. Some of these candidates gave an exact answer which was then followed by the decimal equivalent. This was condoned as an exact answer had been seen. Those candidates who gave a decimal answer only, however, were unable to gain any credit.

(iv) Unfortunately, it was again fairly common to see solutions that used inexact values for e^2 and hence a decimal answer for *x* rather than an exact value of x as required by the question.

This question highlights the importance of candidates to understand and appreciate the meaning of the word 'exact' when used in a mathematical context.

Question 9

(i) Most candidates obtained the equation $2\cos 3x = 1$ and were thus able to find a value for 3*x*. Some candidates then went astray in finding correct results for *x*. Many candidates then considered positive values only for *x* apparently not realising that *P* had a negative coordinate. The diagram had been given in order to help candidates decide on the appropriate values for x, but it was clear that many candidates had completely discounted the diagram. Many candidates offered solutions of $\frac{\pi}{9}$ and $\frac{5\pi}{9}$ which did earn them the first two available marks.

(ii) Most candidates attempted integration with many correct attempts seen. The most common error was to have the coefficient of sin3x as 6. As the solution was required in exact form a method mark was available if the substitution of the candidate's limits was seen still in exact form, but many candidates went straight to a non-exact form. There were a number of candidates who did not make it clear which limits they were using in their work. Only a minority of candidates used the subtraction method which yielded the integrand $2\cos 3x - 1$. Most candidates considered the area of a rectangle either calculated using integration or by a simple application of length multiplied by width. Occasionally this area was omitted.

Question 10

(i) This question required candidates to have a clear understanding of the surface area and the volume of a solid. Most candidates made a good attempt at the required proof, realising that they needed to write formulas for both volume and surface area and eliminate *h*, the height, between the two formulae. Many candidates managed this very efficiently, but others were less careful with their algebra. Many considered the container to be closed when considering the surface area and, as a result, many ended up with a term of $8x^2$ rather than the required $4x^2$. This was often corrected properly, but too many just 'obtained' the given result. It was essential that candidates write down that they were considering the surface area as some just wrote down terms which were not equated or allocated to surface area.

(ii) The majority of candidates recognised this as a maximum/minimum question and knew the process needed. Most candidates were able to differentiate the given equation correctly and then find *x* when $\frac{dS}{dx} = 0$ with few errors, although there were some who were unable to deal with $\sqrt[3]{250}$ correctly. Too many candidates did not find the value of *S* for often a correct value of *x*, highlighting the importance of checking that the demands of the question have been met. The correct use of the second derivative and subsequent conclusion of a minimum was quite common; however, it was essential that any calculations used were correct. A few candidates appeared to consider the gradient on either side of the turning point, but rarely presented their evidence in sufficient detail to make a convincing argument. A few candidates were unclear about the whole process and attempted to find a minimum value by erroneously setting the second derivative to zero and finding the value of *x* for this condition.

Question 11

Most candidates were able to use their problem solving skills and formulate a correct approach and order of operations to produce a clear solution for this unstructured question. There were very few errors in the differentiation of the product involved which also involved the use of the chain rule. Again, an exact response was needed so it was expected that candidates work with exact values/fractions throughout. Those candidates that did resort to the use of decimals, were able to obtain method marks and were not overly penalised for not working with exact values. There were many completely correct solutions to this question. Some candidates were less clear about the process and after successfully differentiating the product, set the gradient to zero and attempted to find what would have been the value of *x* at a stationary point.

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

Candidates should read each question carefully and identify any key words or phrases. Candidates also need to show enough method so that marks can be awarded. Candidates need to be aware of instructions in questions such as 'Showing all your working…' or 'Show that…'. Such instructions mean that when a solution is incomplete, often through calculator use, a significant loss of marks will result. Candidates should ensure that their answers are given to a greater degree of accuracy than that demanded in a question before they round as required. When no particular accuracy is required, candidates should ensure that they follow the instructions printed on the front page of the examination paper. This is especially the case for angles measured in degrees. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions. When finding angles in radians, it is better to have the calculator in radian mode rather than to find the angle in degrees and then make a conversion.

General comments

Most candidates seemed to be well prepared for this examination and many excellent solutions were seen. Candidates were able to recall and use manipulative techniques when needed. Many candidates were also able to write problems using correct mathematical form. Some candidates may have improved if they had had a better understanding of the necessity to use correct bracketing to ensure correct mathematical form for functions with arguments, such as trigonometric functions and logarithms. This was seen in **Questions 1** and **7(b)(i)** in this examination.

Candidates usually presented their work in a clear and logical manner. Some candidates used additional paper for **Questions 7(b)(i)** and **8(a)**. This ensured that their work was readable and could be marked. Candidates who did this usually added a note in their script to indicate that their answer was written, or continued, elsewhere. This was very helpful.

Showing a clear and complete method for every step in a solution is essential if a question asks candidates to 'Show that…' a result is in a particular form. This instruction indicates that the answer has been given and that the marks will be awarded for the method. The need for this was highlighted in **Questions 3(i)**, **7(b)(i)** and **9(b)(i)** in this examination.

Candidates should also understand that, when a part of a question begins with the word 'Hence...', it is expected that they should use the previous part or parts of the question to answer this part. This will often be the most straightforward method of solution and will be assessing a specific skill. This was seen in **Questions 3(ii)**, **5(ii)**, **6(b)(ii)**, **7(b)(ii)** and **9(b)(ii)** in this examination.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

A good number of candidates were able to differentiate sin*x* correctly. Not as many candidates were able to differentiate $\ln x^2$, with $\frac{1}{x^2}$ frequently seen. A good number of candidates were able to apply the correct form of the quotient rule, with the difference in the numerator correct and the denominator correctly squared. It was important to have correct bracketing in the final answer to ensure that the expression given was unambiguous. Candidates would have improved by writing $\frac{(\ln x^2)(\cos x) - (\sin x) \times \left(\frac{2}{x}\right)}{(\ln x^2)^2}$ as their first line, for example, rather than $\frac{\ln x^2 \cos x - \sin x \times \left(\frac{2}{x}\right)}{(\ln x^2)^2}$, which had ambiguities in both terms of the numerator and did not earn the accuracy mark. A few candidates incorrectly gave the denominator as $\ln x^4$, confusing what was being squared. A small number of candidates attempted to use the product rule. This was much less successful as, often, the rearrangement to a product was incorrect and also the derivative of $(\ln x^2)^{-1}$ was more complex.

Question 2

A good number of fully correct solutions were seen. Most candidates started correctly and formed a correct expression for the discriminant. Those who were sufficiently careful arrived at a two-term quadratic expression which was simple to factorise by extracting the common factor, *k*. A few candidates, even after writing $c = -k$, used $c = 1$ or $c = k$ in their expression. Some candidates were unable to deal with the significant number of minus signs in the expression and errors in simplification were not uncommon. Those who made simplification errors usually had to apply the quadratic formula to find their critical values. Candidates who drew sketches usually obtained a pair of inequalities of the correct form for their final answer.

Question 3

(i) Most candidates used the factor theorem correctly, as required, with many earning full marks. The most efficient process was to solve $a(2^3) - 12(2^2) + 5(2) + 6 = 0$. As the answer was given, sufficient method needed to be shown to ensure that the accuracy mark could be awarded. An interim equation with simplified values, such as $8a - 48 + 10 + 6 = 0$, was required. A few candidates used synthetic division with algebraic expressions, which was allowed as this process uses the root and hence the factor theorem had been applied. A few candidates attempted long division. This was not permitted as, whilst the process itself was not incorrect, it did not answer the question. A few other candidates benefitted from the special case mark available for using $a = 4$ and showing that the result was 0. This was not given full credit as the process had been eased.

(ii) Many candidates were successful here and offered fully correct and complete solutions. Some candidates omitted to state the full factorisation of the cubic expression. These candidates commonly factorised the quadratic factor and then stated all three roots. As the cubic expression was required in factorised form, this was not credited. Occasionally candidates only stated the two roots which arose from the quadratic factor. Some candidates did not factorise but found the roots applying the quadratic formula to the quadratic factor equated to 0. This was not credited as the instruction in the question was clear. The roots were to be found by factorising the cubic expression and then solving. Some candidates ignored the given factor $x - 2$ and used one of the other factors. For this to be credited, candidates needed to justify that the factor they had used was indeed a factor. This was not always seen and some candidates are still too reliant on their calculator for solving cubic and quadratic equations. This was very evident from candidates who composed the incorrect factorisation $(x-2)\left(x-\frac{3}{2}\right)\left(x+\frac{1}{2}\right)$, which was not credited.

Question 4

Many candidates found this question quite challenging. Better candidates determined that the radius was $\frac{x}{2}$ and worked correctly with this, obtaining the correct, exact instantaneous rate of change, which was required as their final answer. These candidates used the correct notation, stated a correct chain rule and worked in terms of π throughout. A few candidates decimalised their answer or worked with decimals, resulting only in the correct decimal form of the answer. This did not earn the accuracy mark as an exact form was required. A few candidates worked with δx and δA rather than $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}A}{\mathrm{d}t}$. Many of these candidates were often able to form a correct chain rule and obtain the correct answer, although others confused themselves because of their incorrect descriptions. Some candidates confused x with r and $A = \pi x^2$ as an incorrect starting point was common. A few candidates were able to form the correct expression for the area in terms of x but were unable to differentiate it correctly, with $\frac{2\pi x}{2} = \pi x$ being very common. A few candidates used $\frac{1}{2}r^2\theta$ for the area. This introduced an unnecessary complication and many of these candidates did not use $\theta = 2\pi$. There were a few attempts to find the average rate of change rather than the instantaneous rate of change. This did not answer the question.

Question 5

(i) Many candidates answered this correctly. A few candidates needed to take more care with brackets as, on occasion, the value of r was incorrect and this was commonly as a result of a bracketing error. A few candidates did not give an expression of the required form. Other candidates did little more than factor 5 out of the first two terms.

(ii) This part of the question assessed the ability of candidates to interpret their expression once they had completed the square. Candidates, therefore, needed to use their answer to part **(i)** to answer this part. Some candidates were able to do this successfully and simply wrote down that the least value was $\frac{1}{5}$ of –10.25, i.e. –2.05, and this occurred when $x = 1.5$. Other candidates used calculus to find the value of x at which the expression was a minimum and substitute. This was only accepted if the values obtained corresponded to the values which followed from part **(i)**. Calculus could have been used as a check, of course, and this may have helped some candidates make corrections to part **(i)**, where needed. Some candidates were unable to correctly identify the values they had found and stated that the least value was 1.5 and it occurred at –2.05, for example. A few candidates restarted in this part and completed the square again, rather than observing that the expression in this part was $\frac{1}{5}$ of the expression in part **(i)**. This was allowed as the required interpretation still needed to be carried out. A few candidates divided 1.5 by 5 as well as dividing –10.25 by 5. This was incorrect as the value of x was unaffected by the transformation made. These candidates could have *checked* this using calculus if they had wished. Weaker candidates tended to find only the roots of the given expression set equal to zero.

Question 6

(a) This part was almost always correctly answered. A few candidates stated 4 by 2 and some simply described the number of rows and columns which was not accepted.

(b) (i) A very good number of fully correct answers were seen. Occasionally, sign slips resulted in the determinant of **A** being incorrect with –2 and 10 both seen on occasion. The adjoint matrix was almost always correct. Occasionally candidates changed the signs of the elements in the leading diagonal and changed the positions of the terms in the other diagonal.

(ii) Again, a good number of fully correct answers were seen. Most candidates used the correct strategy of squaring the inverse matrix found in part **(b)(i)** and earned three marks here very efficiently. Other candidates found the matrix $\mathbf{A}^2$ and then found the inverse of that. This was allowed for full credit on this occasion. Some candidates did not recall the matrix **I** correctly. The matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ was seen and used several times, as was $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, for example. Solutions attempted using simultaneous equations were more complex than necessary and much more prone to error than solutions using the product of inverse matrices, which was expected.

Question 7

(a) A good number of candidates gave fully correct solutions. Many candidates were able to make a correct first step in the method, commonly $x^2 - 3 = 10^0$. Some candidates omitted the negative solution while other candidates stated it and then disregarded it because it was negative. This was not appropriate for this question and candidates should look carefully at what is given before disregarding solutions. A few candidates incorrectly wrote $x^2 - 3 = e^0$, which was not accepted. A few candidates did not interpret the logarithm correctly and wrote $\lg x^2 - \lg 3 = 0$, indicating a total misunderstanding. Other candidates wrote $\frac{\lg x^2}{\lg 3} = 0$, misapplying the subtraction of logs/division of arguments rule.

(b) (i) A reasonable number of fully correct answers were seen. As the answer had, in effect, been stated, full method for each step needed to be seen for marks to be awarded. There were many approaches available to candidates in manipulating the expression given to the required form. Candidates whose solutions were fully correct showed full method for each step and were careful with brackets, ensuring their statements were mathematically correct, as required. Some candidates were able to write the answer down because the required form had been given but showed insufficient working to be credited. Weaker candidates tended to ‘cancel’ $\ln a$ and state the answer as $\sin(2x + 5) + 1$, for example. Many candidates omitted brackets in this question and expressions such as ‘$\sin(2x + 5) - 1\ln a$’ were not accepted. Some candidates attempted to verify that the expression could be written in the required form for particular values of a, such as 1 or e. This did not answer the question and so was not credited. A few, very weak, candidates attempted to differentiate using the quotient rule. These candidates needed to read the question more carefully.

(ii) Most candidates were able to access this part of the question. It should have been clear to candidates from part **(b)(i)** that they needed to integrate an expression of the form $\sin(2x + 5) + k$ in this part. All three marks could be earned provided this was the case and many candidates were able to do so. Those who did not often earned two marks for integrating the sine term correctly or one mark for a reasonable attempt at integrating the sine term.

Question 8

(a) Many complete, neat and fully correct solutions were seen. Those who identified the correct terms from the general term offered concise and simple solutions with very few errors. Candidates who wrote out the full expansion and then selected terms, occasionally made slips in earlier terms that impacted on the accuracy of the terms needed to answer the question. A few candidates worked accurately until the final step and, at this point, either only stated the positive solution or stated both solutions and then disregarded the negative one, incorrectly. It was clear in the question that more than one value was expected and these candidates may have improved by rereading the question before determining their final answer. A good number of candidates were able to find the correct two terms. However, many then went on to form an incorrect equation, usually by multiplying the coefficient of x^3 by 120. These candidates needed to take more care when reading the key information given in the question. Commonly, weaker candidates forgot to include any powers of two in their coefficients. Some candidates did not understand the difference between ‘term’ and ‘coefficient’ and worked with x all through, including it in their final answer. There were occasional misreads or misinterpretations of the question with candidates choosing, for example, the terms in a^3 and a^5 or the 3rd and 5th terms.

(b) (i) This part of the question was well answered with many candidates able to find the correct four terms. A few candidates stopped at the term in x^2. It may be that these candidates misinterpreted the question as being 'as far as the 3rd term'. Almost all expansions were simplified and many were fully correct.

(ii) Some candidates struggled to understand what was needed here. Better candidates understood the need to substitute $x = -0.01$ into their answer to part **(b)(i)**. Many of these found the accurate decimal arising from this process, 0.66688, and then concluded that this rounded to 0.67. The decimal 0.66688 needed to be seen for the accuracy mark to be awarded. Candidates should take care not to offer values they have already rounded or truncated which they then round again to draw a conclusion, as some did. Many candidates used their calculator to find 0.98^{20}, wrote down the long decimal from it and then rounded. This did not answer the question and was not credited. Others found that x should be -0.01 but then wrote $(1 + 2(-0.01))^{20} = 0.667$, which again did not answer the question. Other candidates substituted $x = 0.98$, incorrectly.

Question 9

(a) This part of the question was generally well answered. The majority of candidates used a correct initial strategy and correctly transformed the equation to one in terms of $\cos x$ only. A few candidates used $\cos^2 x - 1$ and these may have improved if they had written $\cos^2 x + \sin^2 x = 1$ and rearranged it, rather than trying to recall the expression for $\sin^2 x$ in terms of $\cos^2 x$ directly. A few candidates needed to take a little more care with rearrangement, as sign errors were occasionally seen. Many candidates, though, were able to form a correct quadratic equation in $\cos x$ and solve it to find the correct values $\frac{1}{3}$ and $-\frac{5}{2}$. Usually, these candidates were able to state the correct pair of solutions for x to an acceptable degree of accuracy. A few candidates needed to take more care to observe the instructions on the front of the examination paper. Angles in degrees should be rounded to one decimal place. Angles which were truncated to one decimal place or rounded to 3 significant figures were, therefore, not accepted. This, in particular, affected the solution in the 4th quadrant. Weaker candidates sometimes wrote $-13\cos x = 1 - 6\sin^2 x$ which then, incorrectly, became $-13\cos x = 6\cos^2 x$.

(b) (i) This was done very well by many candidates. As this question required candidates to 'Show that' a result of a particular form should be obtained, it was very important for any step in the solution to be fully justified. Simply listing relationships on the side of the page did not meet this requirement, unless it was perfectly clear at what stage a relationship had been used and the relationship stated was explicit for the step required, not implicit. The simplest way to ensure that each step was fully justified was to substitute using the correct relationships. Some candidates gave very neat, concise and accurate solutions, replacing $1 + \tan^2 y$ with $\sec^2 y$ and $\tan y$ with $\frac{\sin y}{\cos y}$. This almost always resulted in full marks. A few candidates needed to do a little more work in the denominator as they used $\tan^2 y = \frac{\sin^2 y}{\cos^2 y}$ and then had some rearranging to do before they reached an equivalent point in the solution. This was a little more prone to error. Some candidates decided to square the expression before substituting. This altered the question and was therefore not credited. A few candidates needed to take more care with writing $\frac{\sin y}{\cos y}$ as $\frac{\sin}{\cos} y$ was not uncommon and not acceptable. Many candidates seemed to know that $\tan y \cos y$ is $\sin y$. It was unfortunate that these candidates often did not justify the second step by showing why this was the case. These candidates needed to know that in a 'Show that' question, all steps need to be justified for full credit to be given. Some candidates attempted to 'rationalise' the denominator. This rarely produced a solution of any value. Some other candidates made the error $4\tan y = \frac{4\sin y}{4\cos y}$. Again, a few, very weak, candidates attempted to differentiate using the quotient rule. These candidates needed to read the question more carefully.

(ii) By contrast, this question was poorly answered. A few fully correct solutions were seen, usually from those using their calculator to find the angle in the 4th quadrant directly and then understanding that this was the only possible solution. Many candidates confused themselves by using $\sin^{-1}(0.75)$ to find the angle in the 1st quadrant and included this positive value as an answer. This was clearly incorrect and this confusion could have been avoided. Candidates who insist upon using this method should use a different letter, as some did, for their base angle and should make it perfectly clear what values they are offering for their answer. Some candidates offered answers accurate only to 2 decimal places, with no longer, more accurate, decimal being stated. These candidates needed to take more notice of the instructions, printed on the front of the examination paper that inexact answers, with the exception of angles in degrees, should be given to 3 significant figures. These candidates may have improved if they had written the longer decimal value down before they rounded. Some candidates worked in degrees and then converted to radians. These were rarely sufficiently accurate to score.

Question 10

(a) A very good proportion of candidates found an acceptable form of the correct unit vector. Those few who were incorrect sometimes multiplied by the magnitude, rather than dividing by it, not fully understanding the meaning of the term unit vector. Other candidates were unable to apply Pythagoras correctly to find the magnitude, or only found the magnitude. This, however, was uncommon. A few candidates needed to take a little more care as, on occasion, sign slips were made when composing their answer.

(b) (i) A good number of accurate and concise solutions were seen for this part. The use of a reasonable diagram was helpful to many candidates. Many candidates found $\overrightarrow{AB}$ and then used a correct vector route, either adding $\frac{2}{3}\overrightarrow{AB}$ to $\overrightarrow{OA}$ or adding $-\frac{1}{3}\overrightarrow{AB}$ to $\overrightarrow{OB}$, to find $\overrightarrow{OC}$. Candidates had to interpret the need to find $\overrightarrow{OC}$. A few candidates were unable to do this and gave the vector $\overrightarrow{CO}$, or occasionally $\overrightarrow{AC}$, as their final answer, which was not credited. A few candidates successfully used $\begin{pmatrix} x \\ y \end{pmatrix}$ for $\overrightarrow{OC}$ and then formed a correct equation using $\overrightarrow{AC} = 2\overrightarrow{CB}$. Other candidates attempted this but made no progress as their starting equation was $2\overrightarrow{AC} = \overrightarrow{CB}$, which was an incorrect interpretation of the given ratio. A few candidates attempted to find the midpoint of $\overrightarrow{AB}$, again misinterpreting the information given. A few other candidates chose algebraic methods involving the magnitude of vectors which they used to form various quadratic equations. Success using these, often very complicated, approaches was very limited and rarely did they result in a correct vector.

(ii) A small number of fully correct solutions were seen. Many candidates were able to find a correct, or correct follow through, vector $\overrightarrow{OD}$. Few of these were able to use this to find the correct value of λ. Those candidates who formed the proportions $\frac{OD}{OB} = \frac{1}{\lambda}$ and then used $\lambda\overrightarrow{OD} = \overrightarrow{OB}$ were most successful. Those who rearranged to $\overrightarrow{OD} = \frac{1}{\lambda}\overrightarrow{OB}$ often stated that $\lambda = \frac{1}{4}$. Some candidates misinterpreted the ratio given, commonly as $OD : DB$ is $1 : \lambda$. These candidates usually stated the answer $\lambda = 3$. A few candidates successfully used the magnitudes of $\overrightarrow{OB}$ and $\overrightarrow{OD}$ to find λ. This was given full credit if exact values were used. Candidates using this approach and choosing to use inexact decimal values were not credited.

Question 11

(i) Many candidates seemed to have an understanding of what was required but were unable to give a sufficiently rigorous explanation to score. It was necessary to indicate that the velocity of this particle could not be zero. This could be done by stating exactly that or commenting that the velocity was always greater than 0, for example. Stating that the velocity could never be negative, only, was insufficient as it did not indicate that the velocity could not be zero. Many comments were based around time always being positive and often these also then mentioned that the velocity was always positive, which was credited. However, some did not explicitly mention velocity or the implication of it was far too vague for marks to be awarded. It is important to make explanations explicit and clear and, should justification be needed, to then state that afterwards. Some candidates seemed to be trying to justify the velocity always being positive but, as they never actually stated this, could not be credited. A very common incorrect answer was that the particle was travelling in a straight line. This showed a lack of understanding as candidates seemed to think this meant it was not possible for the particle to change direction. Another common incorrect answer was 'the velocity is always constant'. There were many comments, with their basis in physics, concerning there being a lack of other forces acting on the particle. These comments were not based upon the information given in the question and were not credited.

(ii) The most efficient method of finding the acceleration was to differentiate the displacement, with respect to t, using the chain rule. Candidates who did this usually earned both marks. A few candidates chose to use the quotient rule. These solutions were slightly more prone to error as some candidates differentiated the constant 4 as 1 rather than 0. A few other candidates were unable to recall the correct form of the quotient rule correctly, which also resulted in an incorrect expression. Many candidates attempted to substitute $t = 5$ into the correct expression. However, some candidates changed the sign of their final answer, either thinking that acceleration was not a signed quantity or thinking that acceleration was always positive, perhaps. A few, weaker, candidates did not differentiate and some tried to use *suvat* equations here. A few other candidates needed to take a little more care as it was not uncommon for candidates to write $(t + 3)$ instead of $(t + 1)$ and often this was simple carelessness and not a misread of the question, as the expression was written correctly elsewhere.

(iii) A good number of candidates understood the need to integrate and did so correctly. Some of these candidates omitted to find the value of the constant of integration or stated that it was zero or made sign errors when attempting to find it. A small number of candidates attempted to integrate the correct expression but made a sign error or multiplied by –2, rather than dividing by –2, for example. A few candidates incorrectly thought that $\int 4(t+1)^{-3}\mathrm{d}t = \int 4\mathrm{d}t \times \int (t+1)^{-3}\mathrm{d}t$. Weaker candidates stated that $d = vt$, often stating 'distance = speed × time' also, and then gave their answer as $\dfrac{4t}{(t+1)^3}$.

(iv) Candidates needed to interpret the distance travelled in the fourth second as being the difference in the displacements when $t = 3$ and $t = 4$. A few candidates did this and were sufficiently accurate to earn full marks. Many candidates misinterpreted the question as being the displacement travelled when $t = 4$ only. Another misinterpretation was that the required distance was the difference in displacements when $t = 4$ and $t = 5$. These partially correct approaches earned a method mark as long as sufficient evidence was seen. Some candidates integrated the expression for v again, even though they had already done this work in part **(iii)**. A few candidates completely ignored their answer to part **(iii)** and used $d = vt$, with $t = 4$, in this part. A few other candidates calculated the displacements at $t = 1, 2, 3$ and 4 and summed all 4 quantities.

Question 12

(a) (i) This part of the question was well answered, with many candidates stating the correct range in an acceptable form. A few candidates seemed to misinterpret $x > 0$ as $x \geqslant 1$ and the answer $g \geqslant -5$ was not uncommon from those who were incorrect. Other candidates who were incorrect stated $g > -5$ and it may be that these confused the domains of f and g or that they misevaluated $4(0)^2 - 9$. A few other candidates stated $g \geqslant -9$ or $x > -9$, which was not accepted as x was not appropriate for the range of this function.

(ii) (iii) Please note that due to an issue with part **(iii)**, full marks have been awarded to all candidates for parts **(ii)** and **(iii)** in order that no candidate will be disadvantaged. The published question paper has been amended to remove the issue.

(b) (i) Many candidates were able to state that the functions were reflections of each other in the line $y = x$. A few candidates mentioned reflection but omitted to state the equation of the line or stated that the line was one of the axes or $y = -x$, for example. Some candidates only made comments about the domain of one being equal to the range of the other and did not comment on the geometrical relationship. Other candidates commented that the functions were one-one or that the x-coordinate of one graph was the y-coordinate of the other, but this again was not a description of the geometrical relationship. Some candidates described the relative positional relationship of the two graphs, but these comments were not sufficiently rigorous to be credited. It was common for candidates to state, for example, the graph of the inverse function will be the opposite of the graph of the function.

(ii) A few candidates were able to utilise the domain of h and stated the negative square root of the correct expression as their answer, earning full credit. A very high proportion of candidates were able to earn two marks, most commonly for finding the positive square root rather than the negative, or for not discarding the positive square root when both signs had been considered, or for leaving their final answer in terms of y. A few candidates made a circular argument and ended up with the same function they had started with. These candidates had usually confused themselves after the point where they swopped the variables. Weaker candidates sometimes square-rooted the expression $x^2 - 1$ term by term as a first incorrect step and were unable to recover.